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Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Scholarship 2025 Calculus

Time allowed: Three hours
Total score: 32

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should answer ALL the questions in this booklet.

Pull out Formulae and Tables Booklet S–CALCF from the centre of this booklet.

Show ALL working. Correct answers only will not be sufficient.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–24 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (▨). This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Question	Score
ONE	
TWO	
THREE	
FOUR	
TOTAL	

ASSESSOR'S USE ONLY

$$\sqrt[3]{125x^3 - 5x^2 + 11x - 3} = \sqrt[4]{5^{4x^2 + 12}}$$

- $$z = 2 - \frac{2 + i}{iz}$$

- (c) How many different complex solutions does the equation $z^{2025} = \bar{z}$ have?

Justify your answer.

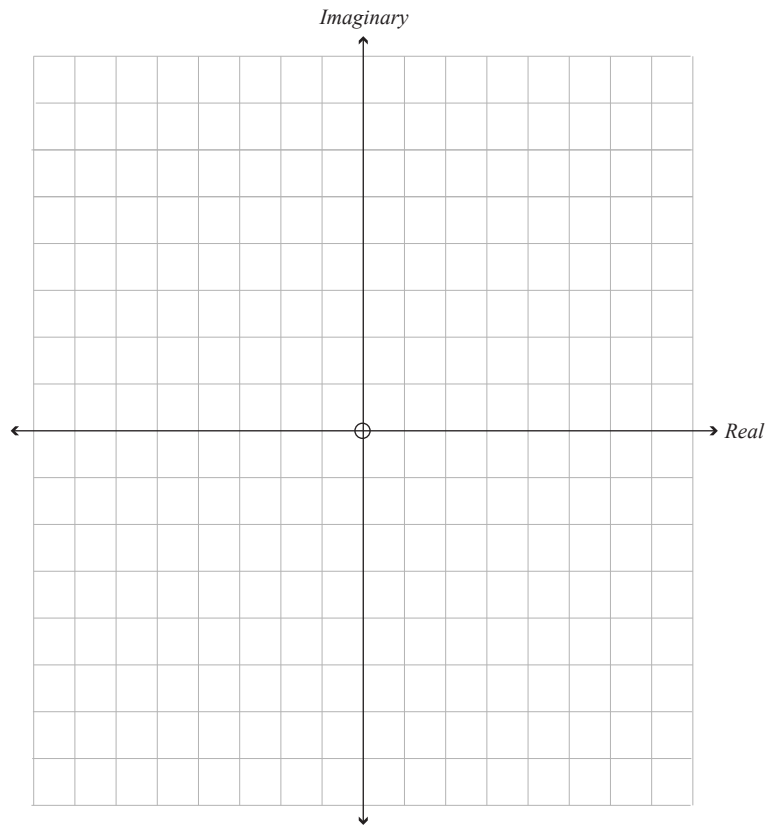
Note: as real numbers and purely imaginary numbers are both subsets of the complex numbers, you should consider them also.

- (d) Let $z = r \operatorname{cis} \theta$ be a complex number with $r > 1$ and $0 < \theta < \frac{\pi}{2}$.

Let the point A represent z in the Argand diagram and the points B, C, and D represent the complex numbers $\bar{z}$, $\frac{1}{z}$, and iz respectively.

If $\angle ABC = 60^\circ$ and $BC = 1.5$, find the area of the quadrilateral ABCD.

Use the Argand diagram below to support your working.



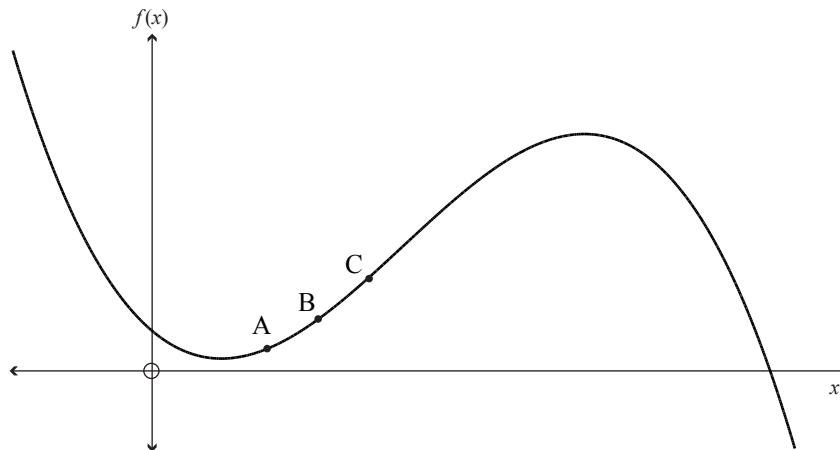
(a) A function is defined parametrically by the equations:

- $p(t) = at^3 + bt^2 + ct + d$
- $p(1) = 2.5$
- $p'(1) = 6$

- Given that $f(1) = 2025$, find the value of $f(1) + f(2) + \dots + f(2025)$.

- (c) Consider the cubic function $f(x) = ax^3 + bx^2 + cx + d$.

The points A(2,4), B(3,9), and C(4,16) all lie on the graph of the cubic, as shown.



Suppose that the lines AB , AC , and BC intersect the cubic again at points D , E , and F , respectively, and that the sum of the x -coordinates of D , E , and F is 25.

Find the y -intercept of the cubic.

Hint: consider the function $g(x) = f(x) - x^2$ in factorised form.

(a) Prove that $\frac{1}{1 + \sin x} = \sec x(\sec x - \tan x)$.

$$\int_0^{\frac{2\pi}{3}} \frac{1}{1 + \sin x} dx$$

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- Find a similar expression for $\tan\left(\frac{3\pi}{8}\right)$.

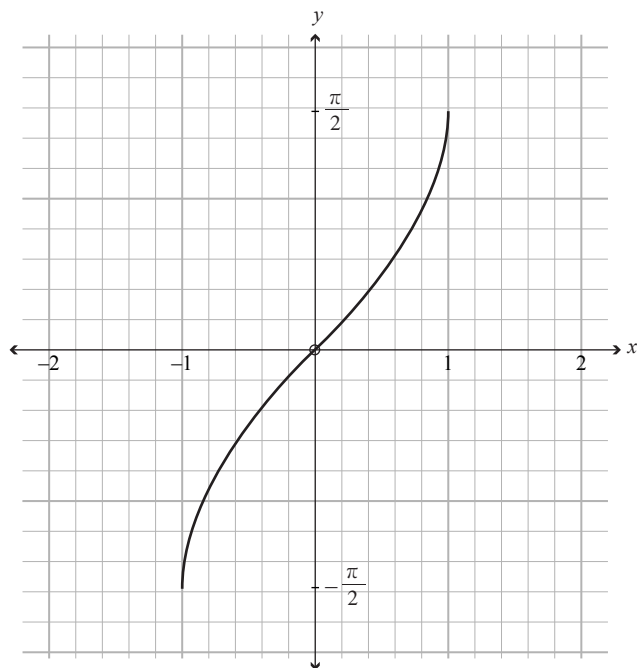
- $$\tan\left(\frac{x}{2}\right)\tan y = \sqrt{2} - 1$$

$$\tan\left(\frac{x}{2} + y\right) = 1 + \sqrt{2}$$

- 1

- 1

(a) The graph below shows $y = \sin^{-1}(x)$, the inverse sine function, on its domain $-1 \leq x \leq 1$.



- $$\int_0^1 \sin^{-1}(x) dx$$

(ii) If $x = \sin y$, show that:

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

Hence, or otherwise, find the minimum value of $g(x) = \sin^{-1}(2x) + \sin^{-1}\left(\frac{\pi}{4} - 2x\right)$.

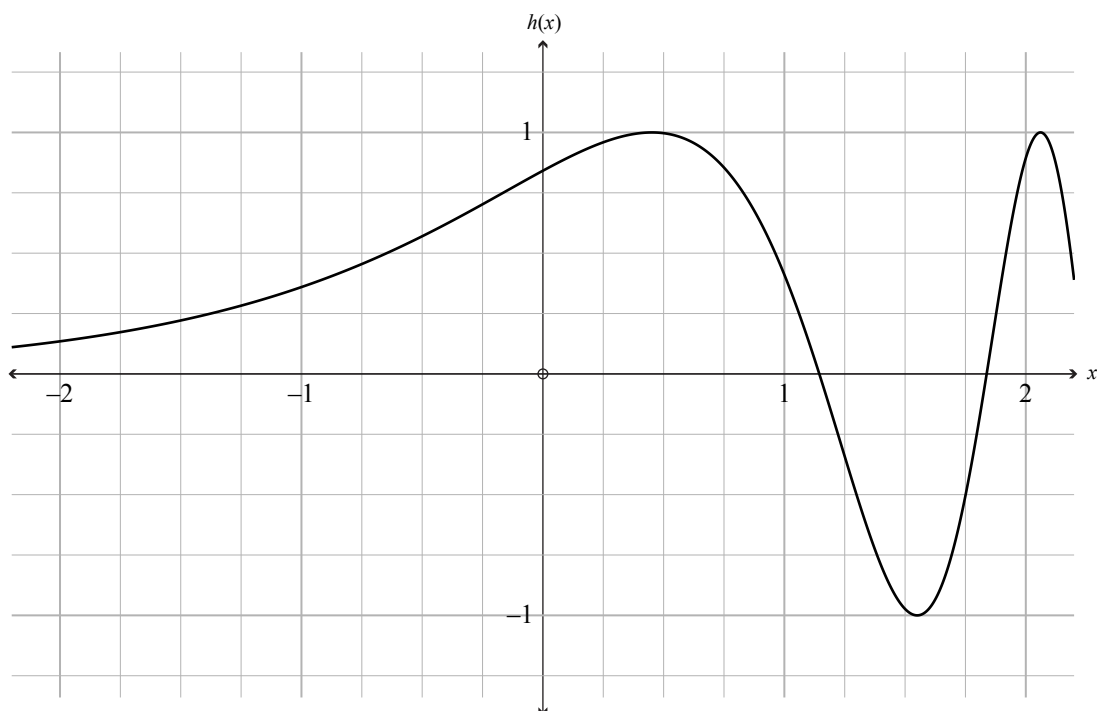
You may assume that your answer is indeed a minimum.

- (b) Consider the function $f(x) = \left(\frac{1}{r^r}\right)^{r+1} \cdot x^r$, where r is a positive, real constant, and $x > 0$.

Find the value of r for which $f(x)$ is a solution of the differential equation $f'(x) = f^{-1}(x)$.

Note: if f and f^{-1} are inverse functions, it can be shown that $f(f^{-1}(x)) = x$ for all x in the domain of f^{-1} .

- (c) The graph below shows part of the graph of $h(x) = \sin(e^x)$.



- (i) Find the value of $\lim_{x \rightarrow -\infty} h(x)$ and $\lim_{x \rightarrow \infty} h(x)$, or state clearly if any do not exist.

Support your answer with mathematical reasoning.

- (ii) Find the exact value of a that generates the maximum value of the definite integral:

$$I = \int_a^{a+1} \sin(e^x) dx$$

You may use the graph above to justify your answer.

**Extra space if required.
Write the question number(s) if applicable.**

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