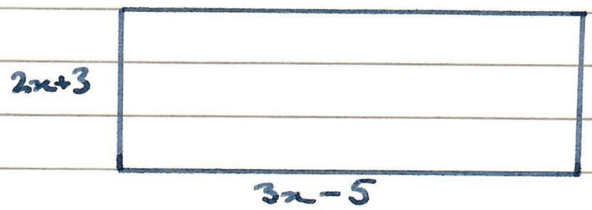
NZQA Level 1 Mathematical Reasoning 2024

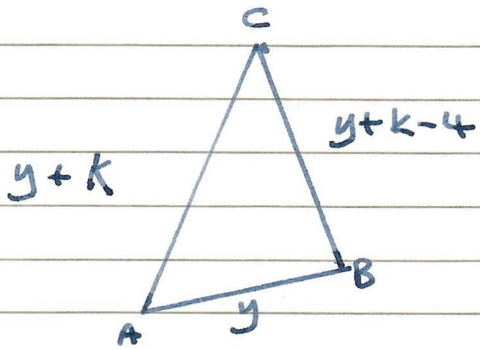
1) a)



Perimeter $= 2(2x+3) + 2(3x-5) = 56$ (given)

$$4x + 6 + 6x - 10 = 56$$
$$10x - 4 = 56$$
$$10x = 60$$
$$\underline{\underline{x = 10}}$$

b)



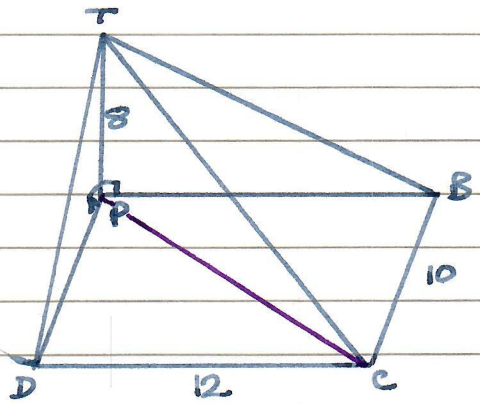
Perimeter $= y + (y+k) + (y+k-4) = 5y$ (given)

$$3y + 2k - 4 = 5y$$
$$2k - 4 = 2y$$
$$y = k - 2$$

And the perimeter $= 5y = 5(k-2)$

$$= \underline{\underline{5k - 10}}$$

c) use Pythagoras twice:



$PC = \sqrt{PD^2 + DC^2}$

$= \sqrt{100 + 144}$

$= \sqrt{244}.$

$TC = \sqrt{TP^2 + PC^2}$

$= \sqrt{64 + 244}$

$= \sqrt{308}$ m $\quad (= 17.55\text{m})$

d) It seems odd that they tell us a sphere is 'like a ball' when the question wants a good understanding of spherical geometry, but hey...

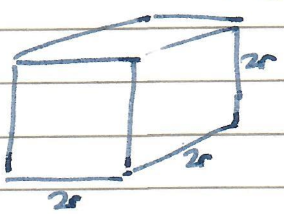
Suspect the ratio of the 'amount per ball' is $4 : \pi$, but here goes:

Let the radius of the sphere be r (since we know $\frac{4}{3}\pi r^3 = 150$ so $r^3 = \frac{200}{\pi}$

– but this doesn't give a tidy number.

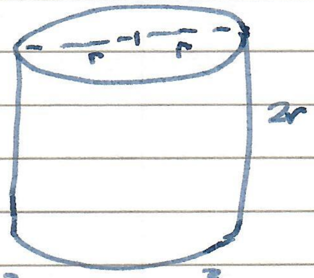
Then the space allocated to each ball:

- in the box is $(2r)^3 = 8r^3$
  so the empty space $= 8r^3 - \frac{4}{3}\pi r^3$



$= (8 - \frac{4}{3}\pi)\, r^3$

- in the cylinder is



$2r.\pi r^2 = 2\pi r^3$

so the empty space is $2\pi r^3 - \frac{4}{3}\pi r^3$

$= \frac{2}{3}\pi r^3$

working out the %s:

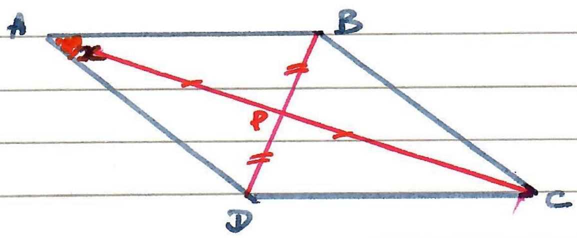
% empty in the box $= \frac{8 - \frac{4}{3}\pi}{8} = 47.64\%$

% empty in the cylinder $= \frac{1}{3} = 33.33\%$

So the box has the greater empty space.

(which was obvious, but now we know the exact numbers.)

2)



Rhombus: Perimeter = 60 cm

So AB etc = 15 cm.

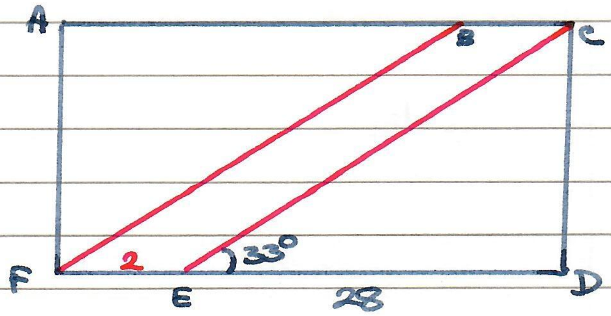
$BP = \frac{BD}{2} = \frac{10}{2} = 5\text{cm}.$

So $\frac{x}{2} = \sin^{-1}\frac{5}{15} = \sin^{-1}\frac{1}{3} = 19.47°$

$x = 38.94°$

(note we have to halve x)

b)



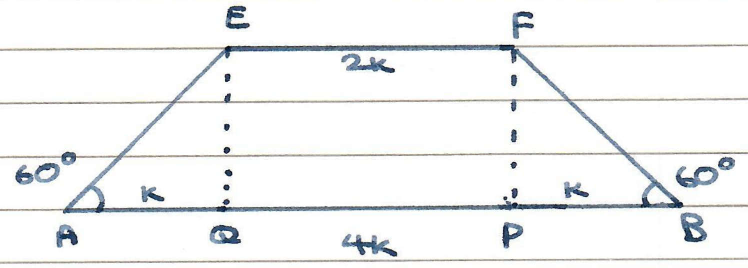
Area of the stripe (it's a ||gram) is $2 \times CD$

and $\frac{CD}{28} = \tan 33^\circ$

$CD = 28 \tan 33^\circ = 18.18$ m

So the stripe area $= \underline{\underline{36.36 m^2}}$

c)



By symmetry, $AQ = PB = k$.

i) $\frac{PF}{k} = \tan 60^\circ = \sqrt{3}$

$PF = \sqrt{3}k = 1.7321k$ m.

ii) Area of the trapezium $= \frac{(EF + AB)}{2} \times FP$

$= \left(\frac{2k + 4k}{2}\right)\sqrt{3}k = 3\sqrt{3}k^2$ m^2

So volume $= (3\sqrt{3}k^2) \times (8k) = 649.519$ (given)

$= 41.569\ k^3$

So $k^3 = 15.625$: $\underline{\underline{k = 2.5}}$

3) a) $y = 5^{3x+2} + 4$

when $x = 0$, $y = 5^{3.0+2} + 4$

$= 5^2 + 4$

$= \underline{\underline{29}}$

b)

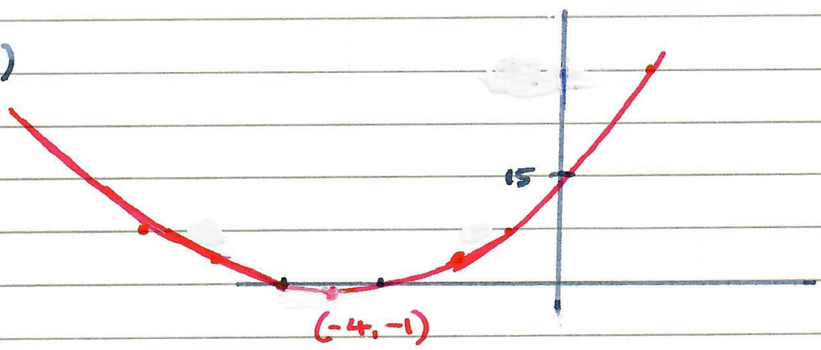
x	y	Δy	$\Delta^2 y$		
1	24			$= 25-1$	5^2-1
2	35	11		$= 36-1$	6^2-1
3	48	13	2	$= 49-1$	7^2-1
4	63	15	2	$= 64-1$	8^2-1
5	80	17	2	$= 81-1$	9^2-1

i) It seems $y(x) = (x+4)^2 - 1$

$= x^2 + 8x + 15$

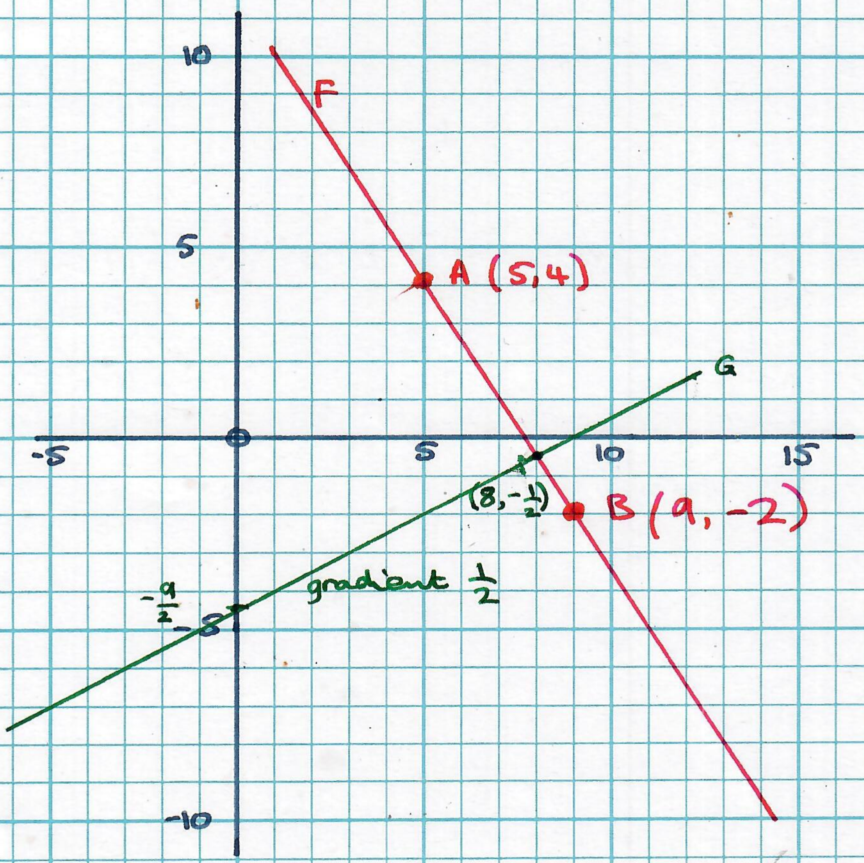
$= \underline{\underline{(x+3)(x+5)}}$

ii)



- Curve is an upward parabola
- Roots are -5 and -3
- Axis of symmetry is $x = -4$
- Minimum is at $(-4, -1)$
- y-intercept is at $(0, 15)$
- Range is $y \geqslant -1$.

c)



(It's nice they say the diagram will 'help' us.)

i) Gradient of the line is

$$\frac{4-(-2)}{5-9} = \frac{6}{-4} = -\frac{3}{2}$$

So the line is $y = \frac{-3}{2}x + c$

Subs. for the point (5, 4):

$$4 = \frac{-3}{2} \times 5 + c$$

$$= -7\frac{1}{2} + c$$

So $c = 4 - 7\frac{1}{2} = 11\frac{1}{2}$

So F is $y = \frac{-3}{2}x + 11\frac{1}{2}$

(Algebraic)

ii) F: $y = -\frac{3}{2}x + 11\frac{1}{2}$ —— ①

G: $x - 2y - 9 = 0$

So $2y = x - 9$

$y = \frac{x}{2} - \frac{9}{2}$ —— ②

Equating ① = ②:

$\frac{x}{2} - \frac{9}{2} = -\frac{3}{2}x + \frac{23}{2}$

$2x = \frac{32}{2} = 16$

$x = 8$

From ②: $y = \frac{8}{2} - \frac{9}{2} = -\frac{1}{2}$

So the intersection is $(16, -\frac{1}{2})$.

(Geometric)

G: $x - 2y - 9 = 0$

$\frac{x}{2} - y = \frac{9}{2}$

$y = \frac{x}{2} - \frac{9}{2}$

This is a line with gradient $\frac{1}{2}$ and y-intercept $-\frac{9}{2}$.

And plotting this line, it meets F at $(8, -\frac{1}{2})$

d) $3^{2x+3} \times 9^{x} = 3^{4-3y}$

Express in base 3:

$3^{2x+3} . 3^{2x} = 3^{4-3y}$

Equate indices:

$2x+3+2x = 4-3y$

$4x+3 = 4-3y$

$3y = 4-3-4x$

$= 1-4x$

$y = \frac{1-4x}{3}$