NZQA Level 1 Mathematical Reasoning 2023

1) a)

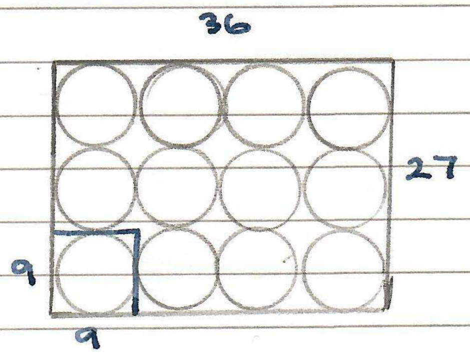
$$T = \pi \sqrt{\frac{h \sin x}{g}}$$

$$= \pi \sqrt{\frac{2.5}{9.81} \sin 75^\circ}$$

$$= \pi \times 0.4961$$

$$= \underline{\underline{1.5587}}$$

b) i)



Since there are 3x4 rows of tins, each takes up 9x9 cm.

So the radius of each tin is 4.5cm.

So the circumference of each tin is $2\pi \times 4.5$
$= 9\pi$ cm.

So area of each label $= 9\pi \times 15 \text{ cm}^2$ (15cm deep)

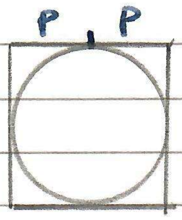
$= 135\pi \text{ cm}^2$

So total area of the labels $= 12 \times 135\pi \text{ cm}^2$

$= 1620\pi \text{ cm}^2$

$= \underline{\underline{5089.38 \text{ cm}^2}}$

ii) The story about it being a different box is a bit of a distraction, but... we can consider just one 'square' space:



In the square: area $= (2p)^2 = 4p^2$
volume $= 15.4p^2$

In the circle: area $= \pi p^2$
volume $= 15.\pi p^2$

So the empty space is $15.4p^2 - 15.\pi p^2$

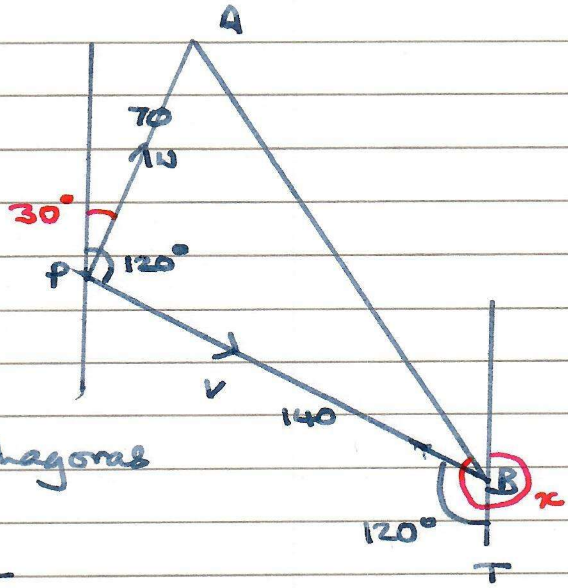
$\therefore$ The proportion of empty space is:

$$\frac{15.4.p^2 - 15.\pi.p^2}{15.4p^2}$$

$$= \frac{4-\pi}{4}$$

And this is the same proportion for each of the 12 small 'boxes' - so the same for the big box.

c) i) This is an easy question dressed up as a hard one...



...the key thing is to notice $\angle APB$ is $90°$.

So we can use Pythagoras to say:

$$AB^2 = \sqrt{70^2 + 140^2}$$

$$= \underline{\underline{156.52 \text{ km}}}$$

ii) $\tan ABP = \frac{70}{140} = \frac{1}{2}$ so $ABP = 26.57°$

and $PBT = 120°$ (corresponding angles)

So the <u>bearing</u> of A from B is

$$180° + 120° + 26.57° = \underline{\underline{326.57°}}$$

(I still don't understand why we make children learn <u>bearings</u> ... unless they're all going to learn sailing.)

iii) Using $s = vt$ i.e. $t = \frac{s}{v}$,

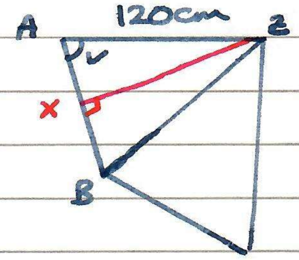
$t_W = \frac{70}{k}$ $\quad t_V = \frac{140}{V_V}$ (V_V = unknown speed of V)

and $t_W + t_V = 4$ (given)

So $\frac{70}{k} + \frac{140}{V_V} = 4$: $\frac{140}{V_V} = 4 - \frac{70}{k} = \frac{4k - 70}{k}$

$$\underline{\underline{V_V = \frac{140k}{4k - 70}}}$$

2) a)



i) In Δ AZB, $8 \times A\hat{Z}B = 360°$

So $A\hat{Z}B = 45°$

So $A\hat{Z}X = 22.5°$ (isoceles Δ with symmetry)

so since AXZ is a right-triangle,

$X\hat{A}Z = v = 90° - 22.5° = \underline{\underline{67.5°}}$ as required.

ii) In Δ AXB:

$\frac{AX}{AZ} = \frac{AX}{120} = \cos 67.5°$: $AX = 120 \cos 67.5°$
$= 45.922\text{cm}$

$\frac{XZ}{AZ} = \frac{XZ}{120} = \sin 67.5°$: $XZ = 120 \sin 67.5°$
$= 110.866\text{ cm}$

So area of Δ AZB $= \frac{1}{2} XZ.AB$

$= \frac{1}{2} XZ . AX . 2$

$= XZ . AX$

So area of the table $= 8.XZ.AX$

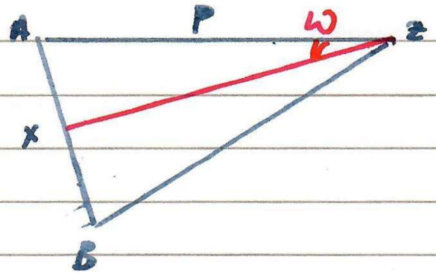
$= 8 \times 110.866 \times 45.922 \text{ cm}^2$

$= \underline{\underline{40\,729.34 \text{ cm}^2}}$

or $\underline{\underline{4.0729 \text{ m}^2}}$

(i.e. quite big)

iii)



The reasoning is similar, though it's simpler to work in terms of the angle

$w = \frac{360^\circ}{2n}$.

Now $AX = p \sin w$

$XZ = p \cos w$

Area $\triangle ABZ = \frac{1}{2} \times 2 \times AX \times AZ$

$= p^2 \sin w \cos w$

So Area of the table =

$n p^2 \sin w \cos w$

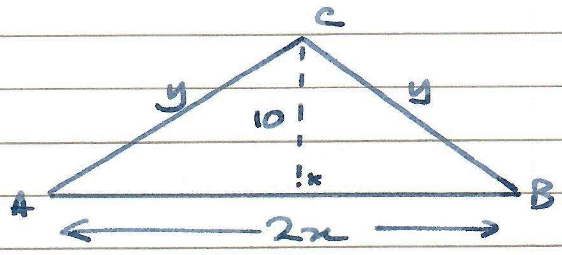
$= np^2 \sin\left(\frac{360^\circ}{2n}\right) \cos\left(\frac{360^\circ}{2n}\right)$

And if we want (the answers apparently do) we can cast these in terms of the v angles by saying

$w = 90^\circ - v$: so $\sin w = \cos v$
$\cos w = \sin v$

and $v = 90^\circ - \frac{360^\circ}{2n}$ etc.

(Published answer seems over-complicated) b)



i) Perimeter ABC = $2y + 2x = 100$ cm (given)

So $y = 50 - x$

i) using Pythagoras in Δ AXC

$$y^2 = 10^2 + x^2$$

But we know:

$$y = 50 - x.$$

So $(50 - x)^2 = 10^2 + x^2$

$$2500 - 100x + \cancel{x^2} = 100 + \cancel{x^2}$$

$$2500 - 100 = 100x$$

$$100x = 2400$$

$$x = 24.$$

So Area of Δ ABC

$$= \frac{1}{2} \times 2x \times 10$$

$$= 24 \times 10$$

$$= 240 \text{ cm}^2$$

3) a)

i)

x	y
1	20
2	25
3	30
4	35
5	40

Clearly $y = 5x + 15$ $= 5.1+15$, $5.2+15$, $5.3+15$, $5.4+15$, $5.5+15$.

ii)

x	y	Δy	$\Delta^2 y$	$2x^2$	$y-2x^2$	$2x$	$y-2x^2+2x$
1	0			2	-2	2	0
2	4	4		8	-4	4	0
3	12	8	4	18	-6	6	0
4	24	12	4	32	-8	8	0
5	40	16	4	50	-10	10	0

a $2x$ term

This means a $2x^2$ term.

So we see $y - 2x^2 + 2x = 0$ in every case

So $y = 2x^2 - 2x$.

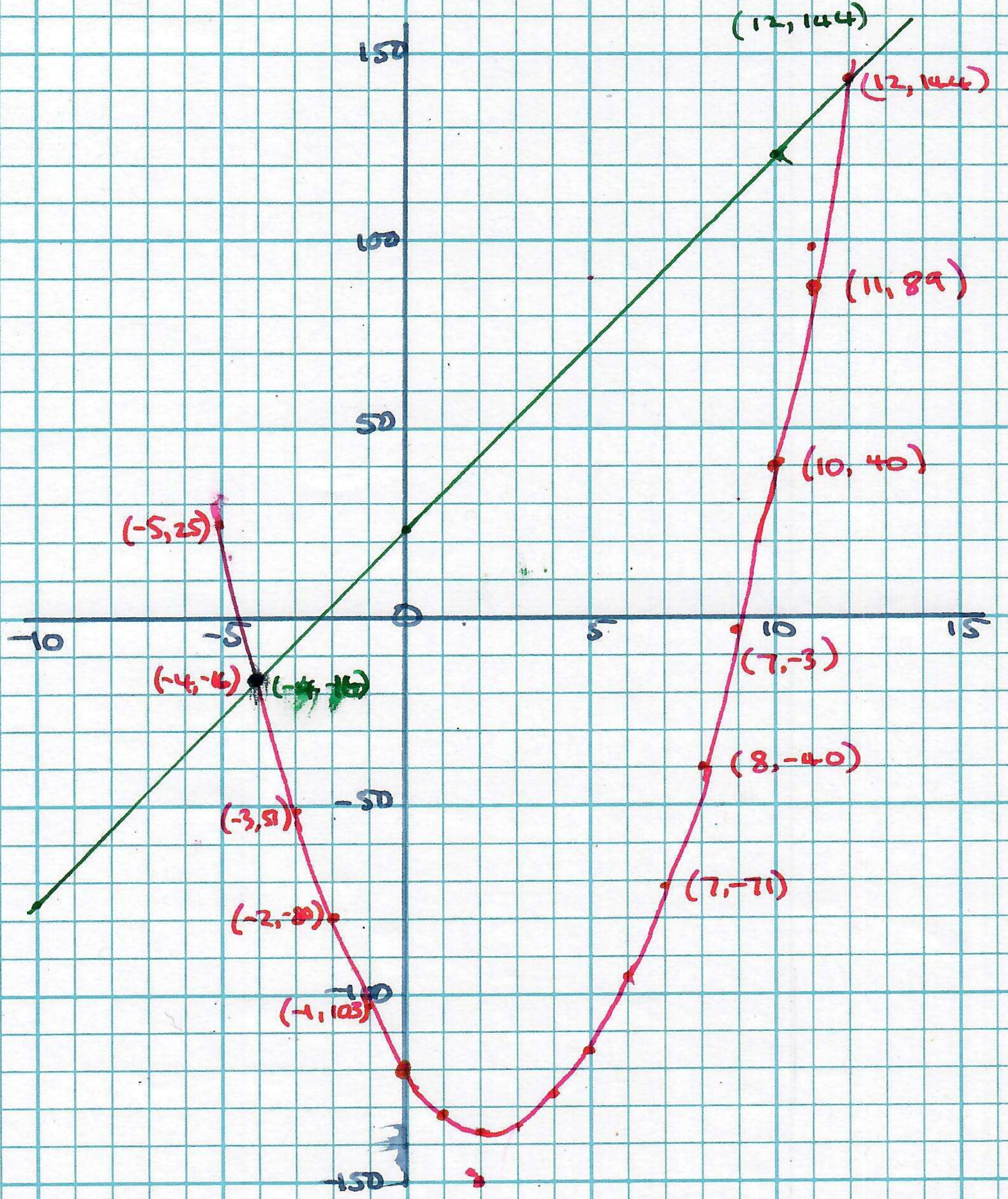
(children are taught this rather by rote, which is probably a good way to tackle these at Level 1 - i.e. by the sequence of thinking laid out here.

iii) where the graphs meet:

$$5x + 15 = 2x^2 - 2x$$
$$2x^2 - 7x - 15 = 0$$
$$(2x+3)(x-5) = 0$$

So $x = -\frac{3}{2}$ or 5

b)

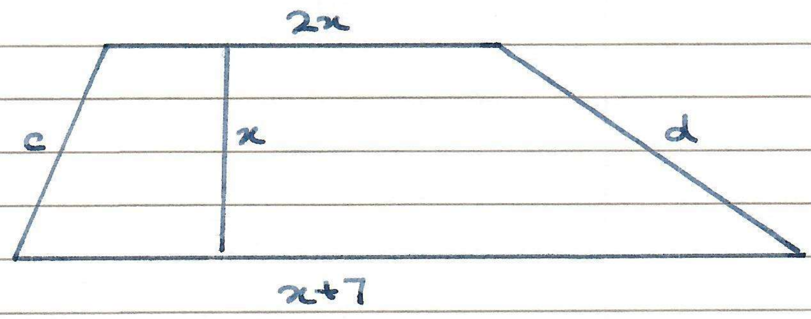


What a ~~disgustingly vile~~ question.
It could have been easier if the parabola had had a nice symmetry - or if they'd scaled it all down - but this is just calculator (or brain) bashing.

Still, the answers come out as (-4, -16)
and (12, 144)

Note this is a ~~pilot~~ paper, and this question was clearly set by someone using Excel spreadsheet.

c) And then a stupidly stupid question...



Ignore c and d – they're just a distraction.

Area $= \dfrac{(2x + (x+7))}{2} \times x = 20\,m^2$ (given)

So $(3x+7)x = 40$

$3x^2 + 7x - 40 = 0$

$(3x-8)(x+5) = 0$

$x = -5$ (discard)

or $\underline{\underline{x = \dfrac{8}{3}}}$