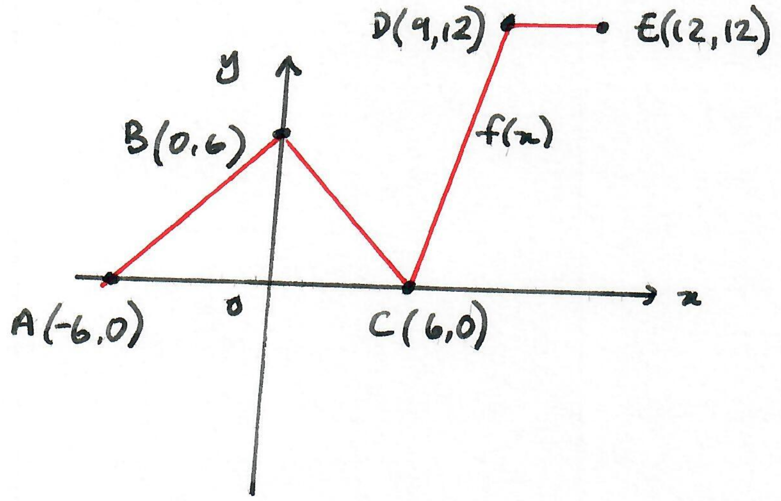
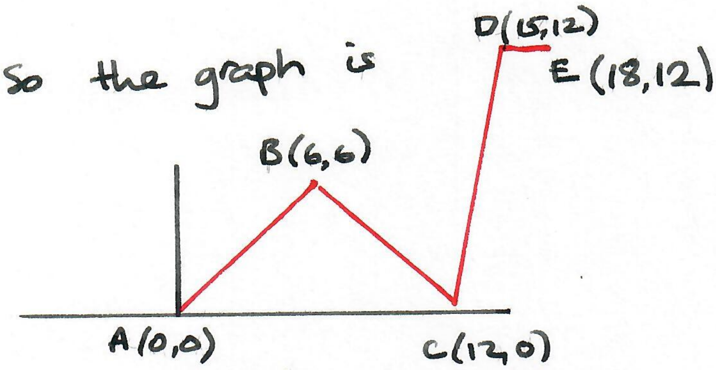
1)

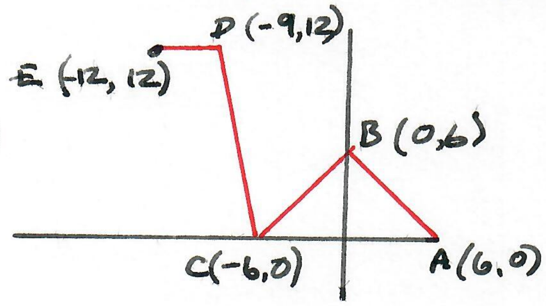


a) $f(x-6)$ calls on $(x-6)$ being 6 left of x

So the graph is

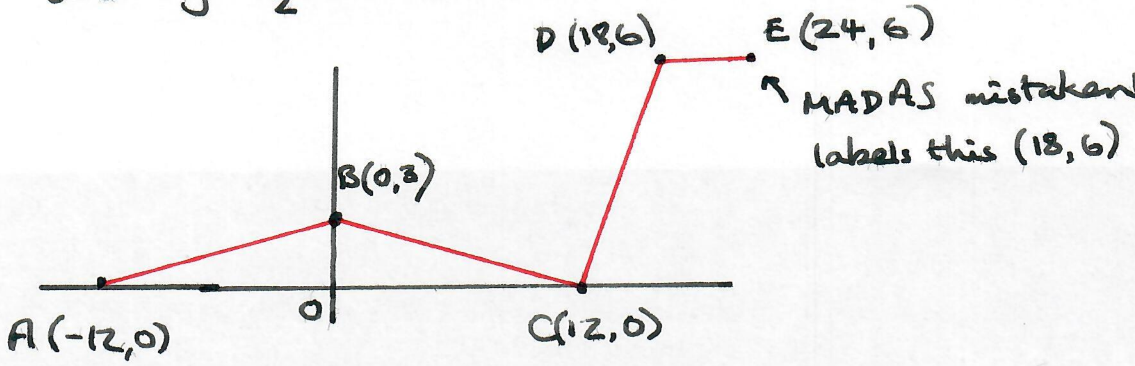


b)



c) $f(\frac{1}{2}x)$ is a $2\times$ stretch on the x-axis

and $y = \frac{1}{2}f(x)$ is a $2\times$ shrink on the y-axis



2) $f(x) = x^3 + 3x^2 - 24x + 20$

a) $(x - \alpha)$ is a factor if $f(\alpha) = 0$, so

$f(1) = 1 + 3 - 24 + 20 = 0$

This shows $(x-1)$ is a factor.

b) By inspection,

$x^3 + 3x^2 - 24x + 20$

$= (x-1)(x^2 + 4x - 20)$

c) $\underline{\underline{x = 1}}$ is one solution: others are given by the quadratic formula:

$$x = \frac{-4 \pm \sqrt{16 + 4 \cdot 1 \cdot 20}}{2}$$

$$= \frac{-4 \pm \sqrt{96}}{2}$$

$$= -2 \pm \sqrt{24}$$

$$= \underline{\underline{-2 \pm 2\sqrt{6}}}$$

These are the other 2 roots.

d) We're now interested in the equation:

$$x^3 + 3x^2 - 24x + 20 = -8$$

i.e. $x^3 + 3x^2 - 24x + 28 = 0$

We know there is a double root at $x = 2$,

so $(x-2)^2 = x^2 - 4x + 4$ is a factor.

By inspection,

$$x^3 + 3x^2 - 24x + 28$$

$$= (x^2 - 4x + 4)(x + 7)$$

So $x+7$ is the third factor and $x = -7$ is the third root

So P is $(-7, -8)$

And as a check, $f(-7) = -343 + 147 + 168 + 20$

$= -8$ as expected.

3) $x = \ln(\sec 3y)$

Approach ①: find $\frac{dx}{dy}$.

$$\frac{dx}{dy} = \frac{1}{\sec 3y} \cdot \frac{d}{dy}(\sec 3y)$$

$$= \frac{1}{\cancel{\sec 3y}} \times 3\cancel{\sec 3y}\tan 3y$$

$$= 3\tan 3y$$

$$= 3\sqrt{\sec^2 3y - 1} \quad \text{and since } e^x = \sec 3y$$

$$= 3\sqrt{e^{2x} - 1}$$

So inverting, $\dfrac{dy}{dx} = \dfrac{1}{3\sqrt{e^{2x}-1}}$

Approach ② find $\dfrac{dy}{dx}$

$$x = \ln(\sec 3y)$$

$$e^x = \sec 3y$$

$$3y = \text{arc}\sec(e^x)$$

$$y = \tfrac{1}{3}\text{arc}\sec(e^x)$$

hmm... I was sure I've done this, but I can't find it in my notes.

We could do it if we knew $\frac{d}{dx}(\text{arc}\sec)$, but this isn't in the list of standard formulae.

However, the internet tells us

$$\frac{d}{dx}(\operatorname{arc\,sec} x) = \frac{1}{|x|\sqrt{x^2-1}}$$

So in this case

$$y = \frac{1}{3}\operatorname{arc\,sec}(e^x)$$

$$\frac{dy}{dx} = \frac{1}{3}\,\frac{1}{|e^x|\sqrt{e^{2x}-1}}\cdot e^x$$

Since e^x is always positive, this gives

$$\frac{dy}{dx} = \frac{1}{3\sqrt{e^{2x}-1}}.$$

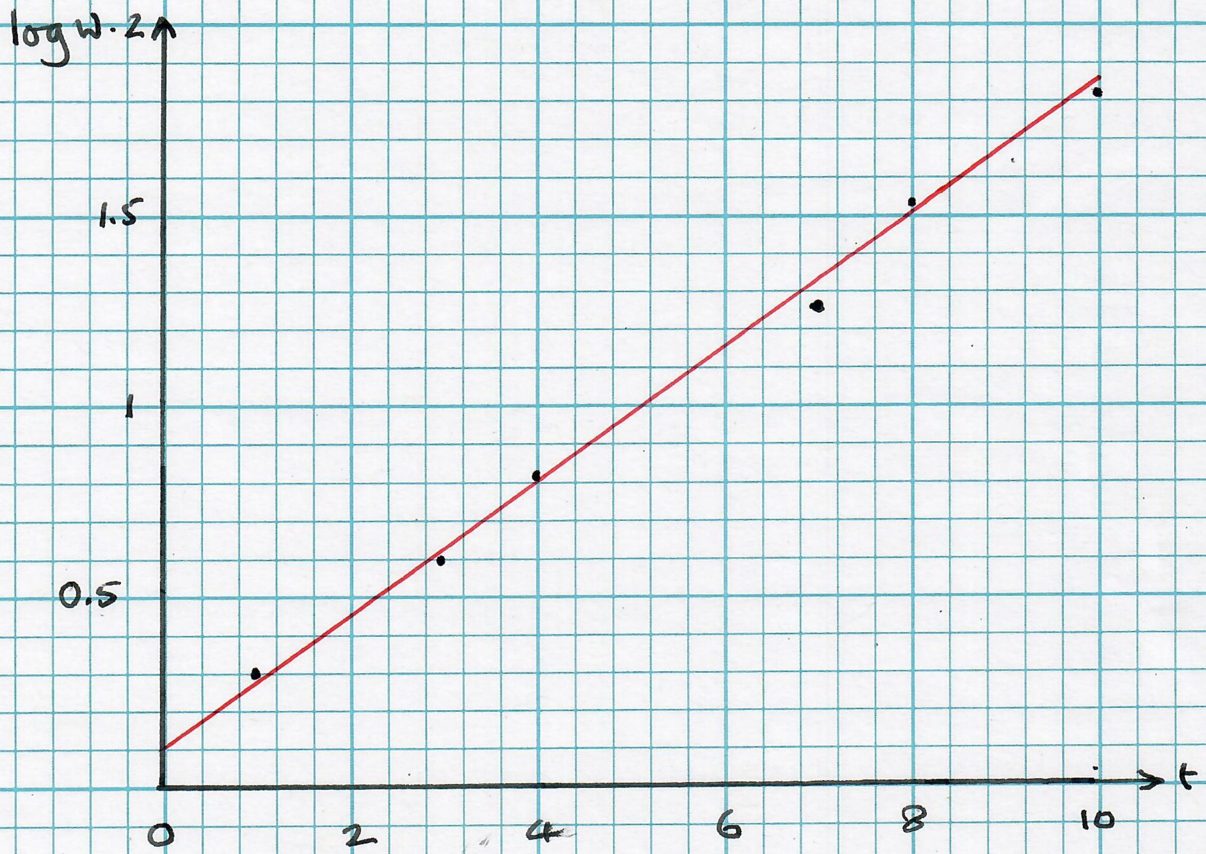
4) a) We're asked for a straight line but the data clearly doesn't follow a straight line. So presumably we will use $W = ab^t$ and logarithms:

$$\log W = \log(ab^t) = \log a + t\log b$$

This is a straight line as required

Find the datapoints:

t	1	3	4	7	8	10
W	2.0	4.0	6.5	19.0	34.0	65.0
log W	0.301	0.602	0.813	1.279	1.531	1.813



The red line intercepts the y-axis at $(0, 0.1)$

and has gradient $\dfrac{(1.85 - 0.1)}{10} = 0.175$

So its equation is $\log W = 0.175t + 0.1$

Comparing this to $\log W = t \log b + \log a$

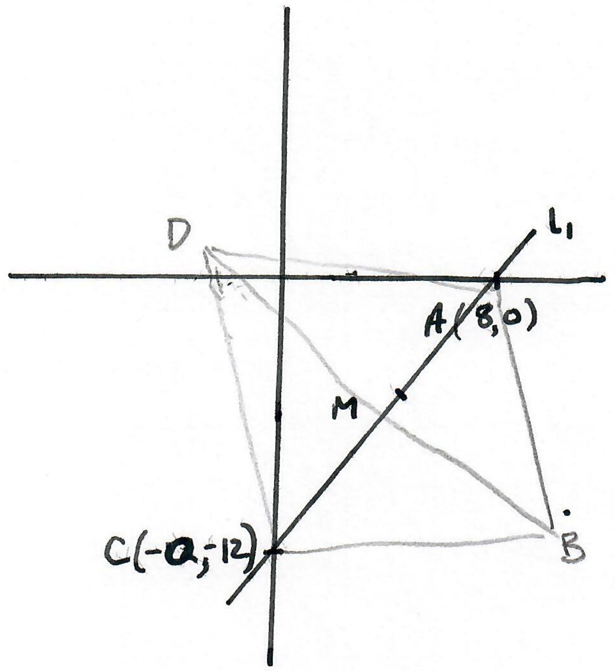
So $\log a = 0.1 \qquad a = 10^{0.1} = 1.26$

$\log b = 0.175 \qquad b = 10^{0.175} = 1.50$

d) Assuming the straight line property still holds when $t = 20$, this gives

$W = 1.26 \times 1.5^{20} = 4190$ (I think MADAS is wrong)

5)



Gradient of l_1 is $\frac{0-(-12)}{8-0} = \frac{12}{8} = \frac{3}{2}$

So gradient of l_2 is $\frac{-2}{3}$ (perpendicular to l_1)

So line l_2 is $y = \frac{-2}{3}x + c$ —— ①

M, midpoint of AC, is $(4, -6)$

Subs. into ①:

$$-6 = \frac{-2}{3} 4 + c$$

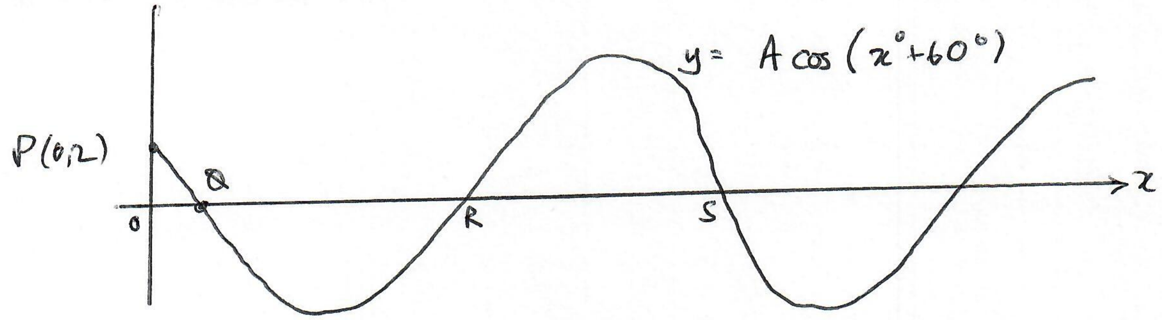
$$\frac{-18}{3} + \frac{8}{3} = c$$

$$c = \frac{-10}{3}$$

So l_2 is $y = \frac{-2}{3}x - \frac{10}{3}$

(slightly surprising that we're not asked to find B and D)

6)



a) At P, $2 = A\cos(0° + 60°) = A\cos 60°$

$= A \times \frac{1}{2}$

So $A = 4$

and the curve is $y = A\cos(x° + 60°)$

b) the intercepts for $\cos(x°)$ would be

$x = 90°, 270°, 450°$

so with $60°$ added, the intercepts are

$x = 30°, 210°, 420°$

and $y = 0$ for each of these.

7) $y = e^{-2x} + axe^{-2x}$

$$\frac{dy}{dx} = -2e^{-2x} + a\left(-2xe^{-2x} + e^{-2x}\right)$$

$$= e^{-2x}\left(-2 - 2ax + a\right)$$

for a stationary point, this is 0.

So $-2 - 2ax + a = 0$ $\quad 2ax = a - 2$

$$x = \frac{a-2}{2a}$$

$$\frac{dy}{dx} = -2e^{-2x} - 2axe^{-2x} + ae^{-2x}$$

$$\frac{d^2y}{dx^2} = 4e^{-2x} - 2a\left(-2xe^{-2x} + e^{-2x}\right) - 2ae^{-2x}$$

$$= (4 + 4ax - 2a - 2a)\,e^{-2x}$$

$$= (4 + 4ax - 4a)e^{-2x}$$

But $x = \frac{a-2}{2a}$

So $$\frac{d^2y}{dx^2} = \left(4 + \frac{\cancel{4a}^{\,2}(a-2)}{\cancel{2a}} - 4a\right)e^{-2\left(\frac{a-2}{2a}\right)}$$

$$= (4 + 2a - 4 - 4a)\;e^{\left(\frac{2}{a}-1\right)}$$

$$= \underline{-2a\,e^{\frac{2}{a}-1}}$$ as required.

8) $\sin(A+B) = \sin A\cos B + \cos A \sin B$

$\sin 3x = \sin(x+2x) = \sin x\cos 2x + \cos x \sin 2x$

$= \sin x(\cos^2 x - \sin^2 x) + \cos x(2\sin x\cos x)$

$= \sin x\cos^2 x - \sin^3 x + 2\sin x\cos^2 x$

$= 3\sin x\cos^2 x - \sin^3 x$

$= 3\sin x(1 - \sin^2 x) - \sin^3 x$

$= 3\sin x - 4\sin^3 x$ as required.

Using this in ①:

$$x^2 + 4x + 4 = 0$$

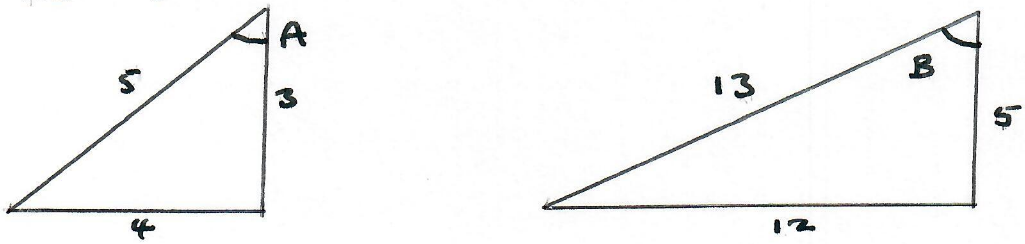
$$(x+2)^2 = 0$$

$$x = -2 \quad \text{(double root)}$$

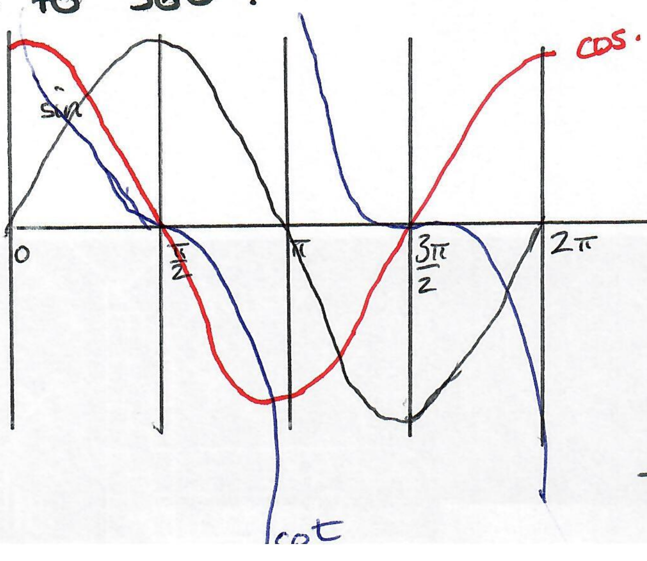
and $y = 2x + 3 = -1$

So the point of contact is $(-2, -1)$

10) Geometrically, there are 2 right-angled Δs here: 3, 4, 5 and 5, 12, 13. This means we can write down the primary values of the trig functions – though we also have to note both A and B are <u>reflex</u> – i.e. $> 180°$.



Now we plot out the relevant functions up to $360°$:



We know A and B are both reflex, with $\cot A < 0$ and $\cos B > 0$, so A and B must both be in the range $\frac{3}{2}\pi - 2\pi$

Trig functions for the reflex angles have the familiar values - but the signs may be different.

From both diagrams:

$$\sin A = \frac{-4}{5} \qquad \cos A = \frac{3}{5}$$

$$\sin B = \frac{-12}{13} \qquad \cos B = \frac{5}{13}$$

Using the trig identities:

$$\sin(A+B) = \sin A \cos B + \sin B \cos A$$

$$= \frac{-4}{5} \cdot \frac{5}{13} + \frac{-12}{13} \cdot \frac{3}{5}$$

$$= \frac{-20-36}{65} = \frac{-56}{65}$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

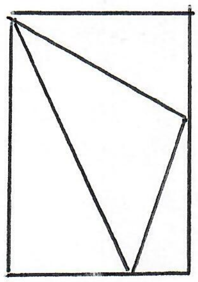
$$= \frac{3}{5} \cdot \frac{5}{13} - \frac{-4}{5} \cdot \frac{-12}{13}$$

$$= \frac{15-48}{65} = \frac{-33}{65}$$

So $\tan(A+B) = \dfrac{\sin(A+B)}{\cos(A+B)} = \dfrac{-56}{65} \times \dfrac{65}{-33}$

$$= \frac{56}{33} \text{ as required.}$$

To find the area of the Δ MADAS divides a rectangle:



and then subtracts the areas of the right-angled triangles.

This is clever, but I'm going to try integration over two parts.

$$\text{Area of } APQ = \int_Q^A (5-x)-(2-3x)\,dx + \int_A^P (5-x)-(3x-2)\,dx$$

$$= \int_{-\frac{3}{2}}^{\frac{2}{3}} 2x+3 \; dx + \int_{\frac{2}{3}}^{\frac{7}{4}} 7-4x \; dx$$

$$= \left[\frac{2x^2}{2}+3x\right]_{-\frac{3}{2}}^{\frac{2}{3}} + \left[7x-\frac{4x^2}{2}\right]_{\frac{2}{3}}^{\frac{7}{4}}$$

$$= \left(\frac{2}{3}\right)^2 + 3.\frac{2}{3} - \left(\frac{-3}{2}\right)^2 - 3\left(\frac{-3}{2}\right) + 7\left(\frac{7}{4}\right) - 2\left(\frac{7}{4}\right)^2 - 7\left(\frac{2}{3}\right) + 2\left(\frac{2}{3}\right)^2$$

$$= \frac{4}{9} + 2 - \frac{9}{4} + \frac{9}{2} + \frac{49}{4} - \frac{98}{16} - \frac{14}{3} + \frac{8}{9}$$

$$= 2 + \frac{12}{9} - \frac{14}{3} + \frac{9}{2} + \frac{40}{4} - \frac{49}{8}$$

$$= 2 + \frac{4}{3} - \frac{14}{3} + \frac{9}{2} + 10 - \frac{49}{8}$$

$$= 12 - \frac{10}{3} + 4\tfrac{1}{2} - \frac{49}{8} = 16\tfrac{1}{2} - \frac{10}{3} - \frac{49}{8}$$

$$= \frac{33.12 - 10.8 - 49.3}{24}$$

$$= \frac{396-80-147}{24} = \frac{169}{24}$$

12) $y = \frac{16}{x^2}$ $\quad y = 17 - x^2$

First find the intersections of the curves - noting there'll be 4 in symmetric pairs and we'll be working with essentially a quadratic in x^2:

Set $\frac{16}{x^2} = 17 - x^2$

$$16 = 17x^2 - x^4$$

$$x^4 - 17x + 16 = 0$$

$$(x^2-1)(x^2-4) = 0$$

So $x = \pm 1$ or ± 4.

Clearly for this question we want to integrate between $x = 1$ and $x = 4$, on the curve $17 - x^2 - \frac{16}{x^2}$

$$\text{Area} = \int_1^4 17 - x^2 - \frac{16}{x^2}\, dx$$

$$= 17x - \frac{x^3}{3} - (-1)\frac{16}{x}\Big|_1^4$$

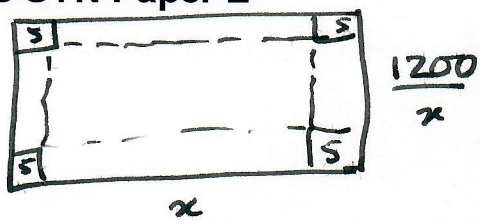
$$= 68 - \frac{64}{3} + 4 - 17 + \frac{1}{3} - 16$$

$$= 39 - \frac{63}{3}$$

$$= 39 - 21$$

$$= \underline{\underline{18}}$$

13)



Volume of the folded box $= 5(x-10)\left(\frac{1200}{x}-10\right)$

and this is to be >2850.

$$5(x-10)(1200-10x) > 2850x$$

$$5(1200x - 12000 - 10x^2 + 100x) > 2850x$$

$$-10x^2 + 1200x + 100x - 12000 - 570x > 0$$

$$-x^2 + 120x + 10x - 1200 - 57x > 0$$

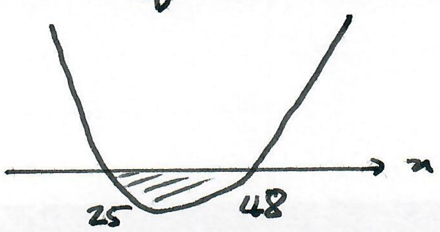
$$-x^2 + 73x - 1200 > 0$$

$$x^2 - 73x + 1200 < 0 \quad \text{as required}$$

It's a beast to factorise, but this works out as

$$(x-48)(x-25) < 0$$

So this has 0s at $x=25$ and 48, and between those values it's negative so satisfies the requirement – see diagram.



14) $$\frac{dy}{dx} = \frac{y^2-1}{x}$$

$$\frac{1}{y^2-1}\,dy = \frac{dx}{x}$$

~~Approach A: Let $y^2-1 = u^2$~~

~~then $2y\,dy = 2u\,du$~~ $\quad dy = \frac{u}{y}du$

~~So $\frac{u\,du}{u\sqrt{u^2-1}} = \frac{dx}{x}$~~

$y+1+y-1$

$\frac{1}{2}(y+1) - \frac{1}{2}(y-1)$

~~$\frac{du}{\sqrt{u^2-1}} = \frac{dx}{x}$~~ $\quad = 0y+1$

~~which doesn't look great.~~

Approach B : separate $\frac{1}{y^2-1}$ into fractions:

$$\frac{1}{y^2-1} = \frac{A}{y-1} + \frac{B}{y+1} = \frac{A(y+1)+B(y-1)}{y^2-1}$$

Equating coefficients:

$0 = Ay + By \qquad A+B=0$

$\qquad\qquad\qquad\quad \cap$

$1 = A - B \qquad A-B=1$

So $A = \frac{1}{2}$ and $B = -\frac{1}{2}$

So we have $\frac{1}{2}\left(\frac{1}{y-1} - \frac{1}{y+1}\right)dy = \frac{dx}{x}$

Integrating, this gives

$$\frac{1}{2}\left(\ln(y-1) - \ln(y+1)\right) = \ln x + c$$

$$\ln\left(\frac{(y-1)}{(y+1)}\right) = 2\ln x + 2c$$

$$= \ln kx^2$$

Note the constant becomes a multiplier, not just a +

where $k = e^{2c}$

So $\dfrac{y-1}{y+1} = kx^2$

Substitute for point $(1,2)$.

$\dfrac{2-1}{2+1} = k.1^2$ So $k = \dfrac{1}{3}$

and the equation is $\dfrac{y-1}{y+1} = \dfrac{x^2}{3}$

Now separate y:

$$3(y-1) = x^2(y+1)$$

$$3y - x^2y = x^2 + 3$$

$$y(3-x^2) = 3 + x^2$$

$$\underline{\underline{y = \frac{3+x^2}{3-x^2}}}$$ as required.

15) Let the series be $a_1, a_2, \ldots$

Then $a_n = a_1 r^{n-1}$

We know: $a_4 - a_1 = 5(a_3 - a_2)$

$$a_1(r^3 - 1) = 5a_1(r^2 - r)$$

$$r^3 - 1 = 5r^2 - 5r$$

$$r^3 - 5r^2 + 5r - 1 = 0$$

$$(r-1)(r^2 - 4r + 1) = 0$$

So $r = 1$ or $r = \dfrac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3}$

So there are 3 possible values of r: $2-\sqrt{3}$, 1, $2+\sqrt{3}$

If a sum to ∞ exists then $|r| < 1$, so the only viable value of r is $2-\sqrt{3}$.

In this case

$$\sum_{1}^{\infty} ar = \frac{a_1}{1-r} = \frac{a_1}{1-(2-\sqrt{3})} = \frac{a_1}{\sqrt{3}-1}$$

This equals $\sqrt{6}+\sqrt{2}$ (given)

So $\dfrac{a_1}{\sqrt{3}-1} = \sqrt{6} + \sqrt{2}$

$$a_1 = (\sqrt{6}+\sqrt{2})(\sqrt{3}-1) = \sqrt{18} + \sqrt{6} - \sqrt{6} - \sqrt{2}$$

$$= 3\sqrt{2} - \sqrt{2} = \underline{\underline{2\sqrt{2}}}$$

So $t = \dfrac{3\pi^{\frac{1}{3}} - \pi . \pi^{-\frac{2}{3}}}{\pi^{-\frac{1}{3}}(2+\pi)}$

$= \dfrac{3\pi^{\frac{2}{3}} - \pi^{\frac{2}{3}}}{2+\pi}$

$= \dfrac{2\pi^{\frac{2}{3}}}{2+\pi}$

So $V = \dfrac{\pi r^2 t}{2} = \dfrac{\pi \pi^{-\frac{2}{3}}}{2} \cdot \dfrac{2\pi^{\frac{2}{3}}}{2+\pi}$

$= \dfrac{2\pi}{2(2+\pi)}$

$= \underline{\underline{\dfrac{\pi}{2+\pi}}}$

17) $x = \frac{1}{2}(-1 + 4\tan\theta)$

a) Substitute:

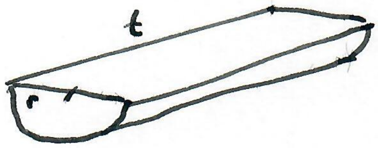
$4x^2 + 4x + 17$

$= (-1 + 4\tan\theta)^2 + 2(-1 + 4\tan\theta) + 17$

$= 1 - \cancel{8\tan\theta} + 16\tan^2\theta - 2 + \cancel{8\tan\theta} + 17$

$= 16\tan^2\theta + 16 = 16(1 + \tan^2\theta)$

16)



Surface area $= \sqrt[3]{27}\,\pi$ i.e $3\sqrt[3]{\pi}$

Surface area is given by

$$\underset{\text{(rectangle)}}{2rt} + \underset{\text{(underside)}}{\pi r t} + \frac{2\pi r^2}{2} = 3\sqrt[3]{\pi}$$

So $t(2r+\pi r) = 3\sqrt[3]{\pi} - \pi r^2$

$$t = \frac{3\sqrt[3]{\pi} - \pi r^2}{r(1+\pi)}$$

Volume $V = \frac{\pi r^2}{2} t$

$$= \frac{\pi r^{\cancel{2}}}{2} \times \frac{(3\sqrt[3]{\pi} - \pi r^2)}{\cancel{r}(2+\pi)}$$

$$= \frac{3\sqrt[3]{\pi}\,\pi r}{2(1+\pi)} - \frac{\pi^2 r^3}{2(1+\pi)}$$

$$\frac{dV}{dr} = \frac{3\sqrt[3]{\pi}\,\pi}{2(1+\pi)} - \frac{3\pi^2 r^2}{2(1+\pi)} = 0 \text{ for a stationary point}$$

So $\cancel{3}\pi^{\cancel{2}} r^2 = \cancel{3}\sqrt[3]{\pi}\,\cancel{\pi}$

$$r^2 = \frac{\sqrt[3]{\pi}}{\pi} = \frac{1}{\pi^{2/3}}$$

So $\underline{\underline{r = \frac{1}{\pi^{\frac{1}{3}}}}}$ or $\sqrt[3]{\frac{1}{\pi.}}$ or $\pi^{-\frac{1}{3}}$

We know $\cos^2\theta + \sin^2\theta = 1$

so $1 + \frac{\sin^2\theta}{\cos^2\theta} = \sec^2\theta$

So $16(1+\tan^2\theta) = 16\sec^2\theta$ as required.

b) $\displaystyle\int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{1}{4x^2+4x+17}\,dx$

Let $x = \frac{1}{2}(-1+4\tan\theta)$

$= \frac{-1}{2} + 2\tan\theta$

then $\frac{dx}{d\theta} = 2\sec^2\theta$

$dx = 2\sec^2\theta\,d\theta$

So the integral is $\displaystyle\int_0^{\frac{\pi}{4}} \frac{2\sec^2\theta\,d\theta}{16\sec^2\theta} = \int_0^{\frac{\pi}{4}} \frac{1}{8}\,d\theta$

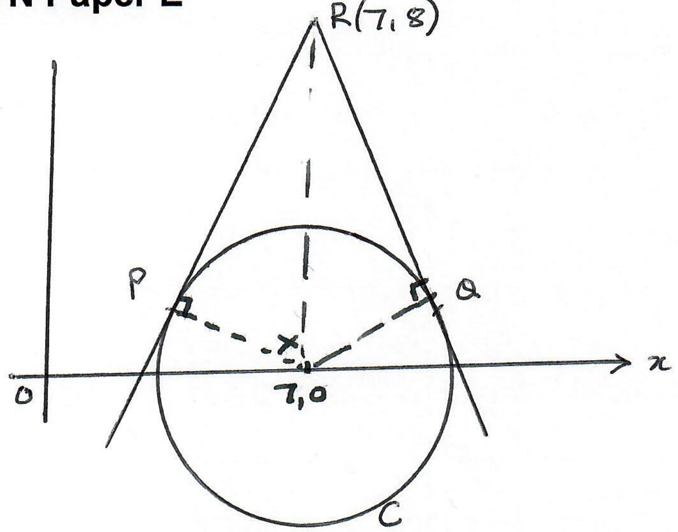
And the limits are

$x = \frac{3}{2}$: $2 = 2\tan\theta$: $\tan\theta = 1$ $\theta = \frac{\pi}{4}$

$x = \frac{-1}{2}$: $0 = 2\tan\theta$: $\theta = 0$

So the integral is $\left.\frac{\theta}{8}\right|_0^{\frac{\pi}{4}} = \frac{\pi}{32}$

18)



a) $C: x^2 + y^2 - 14x + 33 = 0$

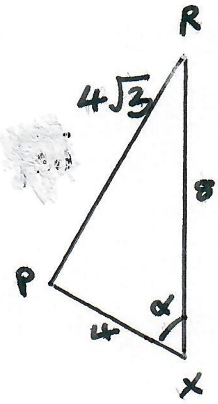
can be expressed as

$$(x-7)^2 + (y-0)^2 - 16 = 0$$

$$(x-7)^2 + y^2 = 16 = 4^2$$

So the centre is (7,0) and the radius is 4.

b) The tangents at P and Q are at right angles to the radii, so we can say for $\triangle$ PRX:



$RX = 8$

$PX = 4$

$PR = \sqrt{8^2 - 4^2} = \sqrt{64-16}$

$= \sqrt{48}$

$= 4\sqrt{3}$

$\angle PXR$: $\cos\alpha = \frac{1}{2}$

So $\alpha = \frac{\pi}{3}$

Area PRX $= \frac{4 \times 4\sqrt{3}}{2}$

$= 8\sqrt{3}$.

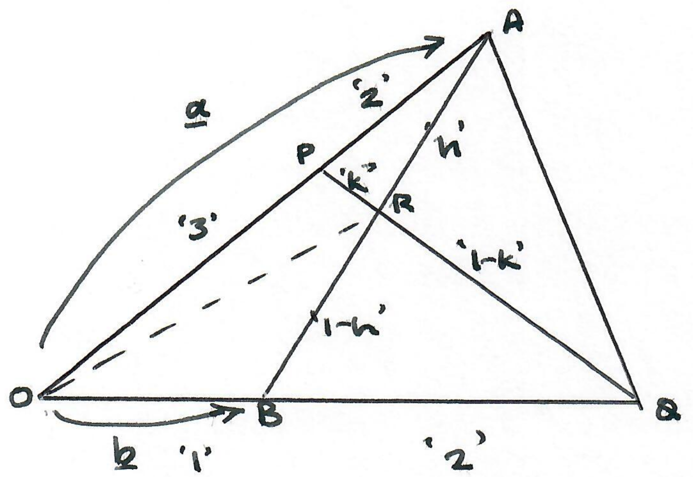
So Area of $PRQX = 16\sqrt{3}$

Area of sector $PQX = \frac{2\pi}{3} \cdot \frac{4^2}{2} = \frac{16\pi}{3}$.

So the shaded area is $16\sqrt{3} - 16\frac{\pi}{3}$

$= \underline{\underline{\frac{16}{3}\left(3\sqrt{3} - \pi\right)}}$ as required.

19)



The question has a couple of catches;

1) the ratio on the A side is $\underline{OP}:\underline{OA}$ (both start at O) whereas on the B side its $OB:\underline{BQ}$

2) similarly $\underline{a}$ and $\underline{b}$ are not given in the same format.

a) Anyway:

$$\overrightarrow{OR} = \overrightarrow{OA} + \overrightarrow{AR}$$
$$= \underline{a} + h\overrightarrow{AB}$$
$$= \underline{a} + h(\overrightarrow{AO} + \overrightarrow{OB})$$
$$= \underline{a} + h(-\underline{a} + \underline{b}) = (1-h)\underline{a} + h\underline{b}$$

as required.

b) $\vec{OR} = \vec{OQ} + \vec{QR}$

$= 3\underline{b} + (1-k)\vec{QP}$

$= 3\underline{b} + (1-k)(\vec{QO} + \vec{OP})$

$= 3\underline{b} + (1-k)\left(-3\underline{b} + \frac{3}{5}\underline{a}\right)$

$= 3\underline{b} - 3\underline{b} + 3k\underline{b} + \frac{3}{5}\underline{a} - \frac{3}{5}k\underline{a}$

$= 3k\underline{b} + \frac{3}{5}(1-k)\underline{a}$

c) Equate expressions for OR:

$$(1-h)\underline{a} + h\underline{b} = 3k\underline{b} + \frac{3}{5}(1-k)\underline{a}$$

Equate coefficients of $\underline{a}$ and $\underline{b}$:

$$1 - h = \frac{3}{5}(1-k) \quad \text{——} \; ①$$

$$h = 3k \quad \text{——} \; ②$$

② in ①: $1 - 3k = \frac{3}{5} - \frac{3}{5}k$

$5 - 15k = 3 - 3k$

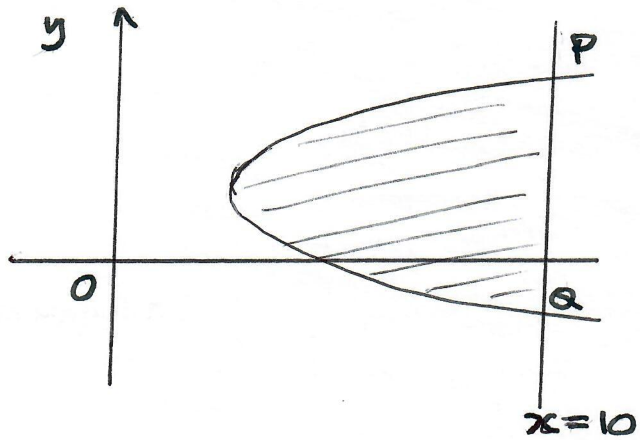
$2 = 12k$

$\underline{k = \frac{1}{6}}$

$\underline{h = \frac{1}{2}}$

Ratio $\frac{PR}{PQ} = \frac{k}{1} = \frac{1/6}{1} = \frac{1}{6}$ or $\underline{\underline{1:6}}$

20)



$$x = t^2+1$$
$$y = 2t+2$$

To me this looks a classic $\int x\,dy$ problem:

MADAS does it two other ways. My approach doesn't get the $8\int_0^3 t^2\,dt$ formulation, though it does get the right answer.

When $x=10$, $t^2+1=10$ so $t = \underset{Q}{-3}$ or $\underset{P}{3}$

$y = \underset{Q}{-4}$ or $\underset{P}{8}$

Also $dy = 2\,dt$.

So the shaded area is

$$2\int_{-3}^{3} 10-(t^2+1)\,dt \quad = 2\int_{-3}^{3} 9-t^2\,dt$$

$$= 2\left[9t - \frac{t^3}{3}\right]_{-3}^{3} = 2(27-9+27-9)$$
$$= 2\times 36 = \underline{72}$$

This is also what $8\int_0^3 t^2\,dt$ gives.

21) a) $\log_2(256x^2) = 1 + 2\log_2\left(\frac{x^4}{2}\right)$

$= \log_2 2 + \log_2\left(\frac{x^4}{2}\right)^2$

$= \log_2\left(\frac{2x^8}{4}\right)$

$= \log_2\left(\frac{x^8}{2}\right)$

So $256x^2 = \frac{x^8}{2}$

$2^9 = 512 = x^6$

$\underline{\underline{x = 2^{\frac{3}{2}} = 2\sqrt{2}}}$

b) $2\log_2\left(\frac{y}{2}\right) + \log_2\sqrt{y} = 8$

$\log_2\left(\frac{y}{2}\right)^2\sqrt{y} = 8$

$\frac{y^{2\frac{1}{2}}}{4} = 2^8 = 256$

$y^{\frac{5}{2}} = 1024$

$y = 1024^{\frac{2}{5}} = 2^{\frac{20}{5}} = 2^4 = \underline{16}$