1) This looks like a simple integral question, though we're not told where A and B are - and interestingly we're not told they're on the x-axis.

So where they intersect:

$$-4x^2+24x-20 = x^2-6x+5$$

$$0 = 5x^2-30x+25$$

$$x^2-6x+5 = 0$$

$$(x-1)(x-5) = 0$$

$$x=1 \text{ or } x=5$$

And by the way

$$y(1) = -4+24-20 = 0$$

$$y(5) = -100+120-20 = 0.$$

so A and B are on the x-axis.

So we need $\displaystyle\int_1^5 -4x^2+24x-20 - (x^2-6x+5)\,dx$

$$= \int_1^5 -5x^2+30x-25\,dx$$

$$= -5\int_1^5 x^2-6x+5\,dx$$

$$= -5\left[\frac{x^3}{3} - \frac{6x^2}{2} + 5x\right]_1^5$$

$$= -5\left[\frac{x^3}{3} - 3x^2 + 5x\right]_1^5$$

$$= -5\left[\frac{125}{3} - 75 + 25 - \frac{1}{3} + 3 - 5\right]$$

$$= -5\left[\frac{124}{3} - 52\right]$$

$$= \frac{-5}{3}(124 - 156)$$

$$= \frac{-5}{3}(-32)$$

$$= \frac{160}{3} = 53.33$$

---

2) $f(x) = |x - 80|$

a) $|x - 80| < 10$

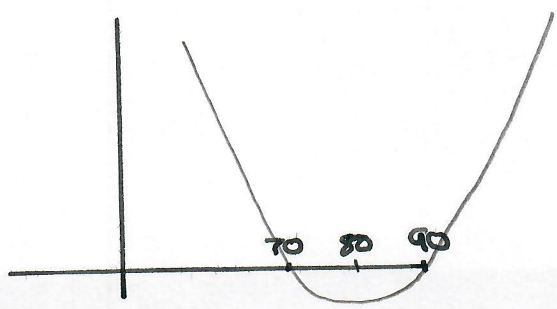
① Square both sides:

$$(x-80)^2 < 100$$

$$x^2 - 160x + 6400 < 100$$

$$x^2 - 160x + 6300 < 0$$

$$(x - 70)(x - 90) < 0$$



So $70 \leq x \leq 90$

b) i) $x - 80 < 0$: then $80 - x < 10$

$$-x < -70$$

$$x > 70$$

ii) $x - 80 > 0$: then $x - 80 < 10$

$x < 90$

So $70 < x < 90$

b) This requires us to find, for some n,

$70 < 1.2^n < 90$

we could do this by finding $\log_{1.2} 70$ etc

but it's easier just to try...

$1.2^{23} = 66.247$

$1.2^{24} = 79.497$

$1.2^{25} = 95.396$

So the required value of n is $\underline{\underline{24}}$.

3) $2x^2 - xy - y^2$

$= (2x \quad y)(x \quad y)$ by inspection

$= (2x + y)(x - y)$ " "

4) $f(x) = \dfrac{4 + x}{(1+3x)^2}$

a) Series expansion of $(1+3x)^{-1}$ is:

$1 + \left(\frac{-1}{1}\right)(3x) + \left(\frac{-1}{2}\right)(3x)^2 + \left(\frac{-1}{3}\right)(3x)^3 + \ldots\ldots$

$= 1 + \frac{-1}{1} 3x + \frac{-1 \times -2}{2} 9x^2 + \frac{-1 \times -2 \times -3}{6} 27x^3 + \ldots$

$$= 1 - 3x + 9x^2 - 27x^3 + \cdots$$

differentiate:

$$\frac{d}{dx}(1+3x)^{-1} = -1(1+3x)^{-2} \cdot 3 = \frac{-3}{(1+3x)^2}$$

$$= -3 + 18x - 81x^2 + \cdots$$

So $\dfrac{1}{(1+3x)^2} = 1 - 6x + 27x^2 \ldots$ as required

c) Series expansion of $f(x)$

$$= (4+x)(1 - 6x + 27x^2 \ldots)$$

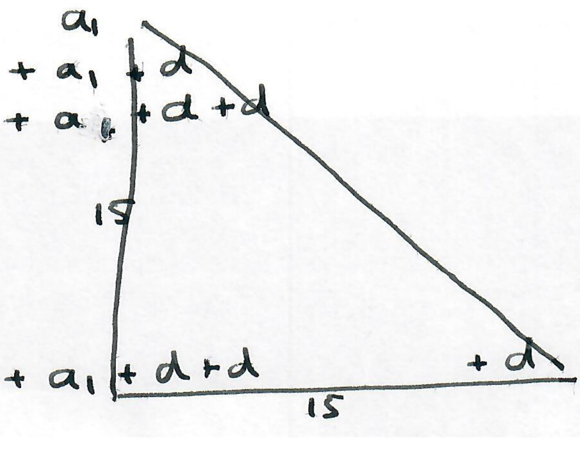
$$= 4 + x - 24x - 6x^2 + 108x^2 + 27x^3 + \cdots$$

$$= 4 - 23x + 102x^2 + \cdots$$

This is all very well, but it doesn't show much sign of converging... and not to the exact answer.

5) $a_{16} = 6 \quad = a_1 + 15d$

$\sum_{i=1}^{16} a_i = 456 =$



$$= ka_1 + \frac{k(k-1)}{2}d$$

So $a_1 + 15d = 6$ —— 1

$$16a_1 + \frac{15.16}{2}d = 456$$

$$16a_1 + 120d = 456 \quad \text{—— 2}$$

1 ×16: $16a_1 + 240d = 96$ 3

3 − 2: $120d = -360$

$$\underline{\underline{d = -3.}}$$

In 1: $a_1 - 45 = 6$

$$\underline{\underline{a_1 = 51}}$$

Sum of k terms is:

$$ka_1 + \frac{k(k-1)}{2}d$$

$$= 51k - \frac{3(k)(k-1)}{2}$$

$$= 102k - 3k(k-1)$$

$$= 102k - 3k^2 + 3k$$

$$= 105k - 3k^2$$

$$= 3k(k-35) = 0$$

So $k = 0$ X or $\underline{\underline{k = 35}}$

6) a) $\frac{dx}{dt} = kx^2$ (where k is negative)

b) First solve the equation:

$$\frac{dx}{x^2} = k\,dt$$

$$x^{-2}\,dx = k\,dt$$

$$-x^{-1} = kt + c$$

Before re-forming this for x, it's worth using this form to find k and c:
(and note x is thousands)

$x(0) = 2.5$: $\quad c = \frac{-1}{2.5} = -\frac{2}{5} = -\frac{16}{40}$

$x(1) = 1.6$: $\quad k - \frac{2}{5} = \frac{-1}{1.6} = \frac{-5}{8}$

$$k = \frac{5}{8} + \frac{2}{5} = -\frac{25+16}{40} = -\frac{9}{40}$$

So $\quad -\frac{1}{x} = -\frac{9}{40}t - \frac{16}{40} = -\frac{9t-16}{40}$

so $\quad x = \frac{40}{9t+16}$ as required.

c) when $x = 250 = .25k$ $\left(\frac{1}{4}k\right)$:

$\frac{1}{4} = \frac{40}{9t+16}$ $\quad 9t + 16 = 160$

$9t = 144$

$t = 16$ months

7) $\left(125^{\frac{1}{3}} \times 25^{\frac{1}{2}} + 16^{\frac{3}{4}} \times 64^{\frac{1}{3}} + \frac{1}{49^{-\frac{1}{2}}}\right)^{-\frac{2}{3}}$

$= (5 \times 5 + 8 \times 4 + 7)^{-\frac{2}{3}}$

$= (25 + 32 + 7)^{-\frac{2}{3}}$

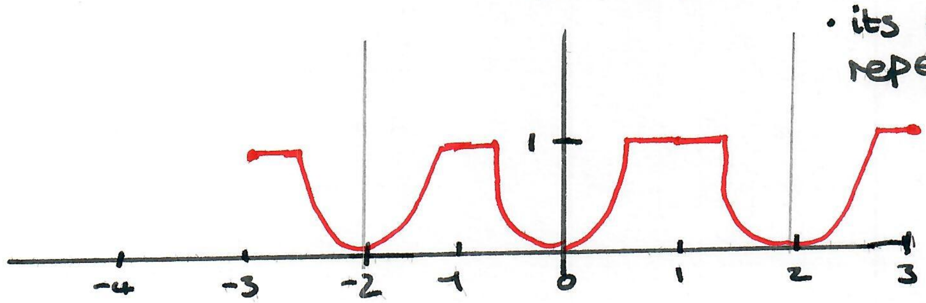
$= 64^{-\frac{2}{3}}$

$= 4^{-2}$

$= \underline{\underline{\frac{1}{16}}}$

8) $f(x) \equiv \begin{cases} 4x^2 & 0 \leqslant x < \frac{1}{2} \\ 1 & \frac{1}{2} \leqslant x \leqslant 1 \end{cases}$

- the function is <u>even</u>, so symmetric about $x = 0$.
- its period is 2, so it repeats every 2.



a) a) A) Do it by differentiating:

$$2x + 2y\frac{dy}{dx} - 10 - 12\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2y-12) = 10 - 2x$$

$$\frac{dy}{dx} = \frac{10-2x}{2y-12}$$

at A(6, 4) this is $\frac{10-12}{8-12} = \frac{-2}{-4} = \frac{1}{2}$

so the gradient of the tangent is $\frac{1}{2}$

and its equation is $y = \frac{1}{2}x + c$.

Substitute for point A: $4 = 3 + c$ $c = 1$.

So the tangent is $\underline{\underline{y = \frac{x}{2} + 1}}$

B) Find the centre of the circle:

i) we 'know' it's (5, 6) from the form
$(-p, -q)$
$x^2 + y^2 + 2px + 2qy + r = 0$

ii) separate as
$$(x-5)^2 - 25 + (y-6)^2 - 36 + 56 = 0$$

$$(x-5)^2 + (y-6)^2 - 5 = 0$$

So the centre is (5, 6), radius $\sqrt{5}$

So gradient of CA is $\frac{6-4}{5-6} = -2$

So gradient of tangent at A is $\frac{1}{2}$, etc.

b) Method A didn't give r, but

Bi gives $\sqrt{5^2+6^2-56} = \sqrt{25+36-56} = \sqrt{5}$

Bii gives $\sqrt{(6-4)^2+(5-6)^2} = \sqrt{2^2+1^2} = \sqrt{5}$.

And since B is $(0, 3)$

$$AB = \sqrt{(6-0)^2+(4-1)^2} = \sqrt{36+9} = \sqrt{45}$$

So since ABC is a right triangle,

$$\text{Area } ABC = \frac{AB.CA}{2} = \frac{\sqrt{45}.\sqrt{5}}{2}$$

$$= \frac{\sqrt{9.25}}{2}$$

$$= \frac{3.5}{2} = \frac{15}{2}$$

$$= \underline{\underline{7\tfrac{1}{2}}}$$

10) $a, ar, ar^2 \;\ldots\; ar^{n-1}$ $\quad r<0$

A) $S_n = \sum_{i=0}^{n-1} ar^i = \frac{a(1-r^n)}{1-r}$ (known)

a) $S_4 = \frac{a(1-r^4)}{1-r}$

$S_2 = \frac{a(1-r^2)}{1-r}$

we know $\frac{S_4}{S_2} = 5 = \frac{(1-r^4)}{(1-r^2)} = 1+r^2$

So $r^2 = 4$

So $r = -2$ (we know $r < 0$).

B) $S_2 = a + ar$

$S_4 = a + ar + ar^2 + ar^3$

$= S_2(1 + r^2)$

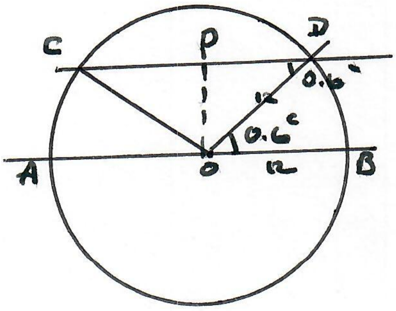
$= 5S_2$

etc.

b) 5th term of the series is $ar^4 = 16a = 36$

So $a = \frac{36}{16} = \frac{9}{4}$ or $\underline{\underline{2\tfrac{1}{4}}}$

11)



c here means 'circular', which means radians - not a use I've come across before.

$\angle COD = \pi - 2 \times 0.6 = \pi - 1.2$

So sector $OCD = \frac{(\pi - 1.2)}{2} 12^2 = (6\pi - 7.2)12$

$\triangle COD = 2 \times \frac{PO \times PD}{2} = \frac{2 \times}{2} 12 \cos 0.6^c \times 12 \sin 0.6^c$

$(0.6^c = 34.38°)$

$\triangledown$ COD $= 139.79$

$\triangle$ COD $= 67.11$

So area of the segment $= 139.79 - 67.11 = \underline{\underline{72.68}}$

Arc CD $= 12(\pi - 1.2) = 23.230$

Line CD $= 12 \times 2 \cos(0.6) = 19.808$

Perimeter $= \underline{\underline{43.038}}$

12) $C: \; y = 4x^2 - 7x + 11$

$L: \; y = 5x + k$

Intersections are given by

$$4x^2 - 7x + 11 = 5x + k$$

$$4x^2 - 12x + (11 - k) = 0 \qquad b^2 - 4ac$$

For there to be 2 solutions the discriminant > 0

i.e. $144 - 4.4(11-k) > 0$

$= 144 - 16.11 + 16k$

$= 144 - 176 + 16k$

$= 16k - 32$

So $16k - 32 > 0$

$16k > 32$

$\underline{\underline{k > 2}}$. as required.

13) $f(x) = h(x)(x-2) + 5$

$f(x) = k(x)(x+2) - 11$

$$f(x) = \underbrace{g(x)(x+2)(x-2)}_{=0 \text{ when } x=-2 \text{ or } 2.} + ax + b.$$

We know $f(a)$ = remainder when div. by $(x-a)$:

So $f(-2) = \quad -2a + b = -11$

$f(2) = \quad 2a + b = 5$

$2b = -6 \qquad b = -3$

$4a = 16 \qquad a = 4$

a) So $f(x) = g(x)(x+2)(x-2)$
$+ 4x - 3.$

Also $f(x) = 3x^4 + px + q$

and we know:
$= g(x)(x+2)(x-2) + 4x - 3$

$= g(x)(x^2-4) + 4x - 3$

Factorising:

$3x^4 + px + q$

$= (x^2-4)(3x^2+12) + 48 + px + q$

So $p = 4$, $q = -51$

and $g(x) = \underline{\underline{3x^2+12}} = \underline{\underline{3(x^2+4)}}$

(and $f(x) = (x^2-4)(3x^2+12) + 4x - 3$)

14) $$I = \int_{\ln 2}^{\ln 5} \frac{3e^{2x}}{\sqrt{e^x - 1}}\,dx$$

Let $u^2 = e^x - 1$ then $2u\frac{du}{dx} = e^x$

$$e^x\,dx = 2u\,du$$
$$e^x = u^2 + 1.$$

So $$I = \int \frac{3(u^2+1) \cdot 2\cancel{u}\,du}{\cancel{u}}$$

$$= 6\int_1^2 (u^2+1)\,du$$

Limits: $x = 5$ $\quad u^2 = e^{\ln 5} - 1 = 5 - 1 = 4 : \; u = 2$

$x = 2$ $\quad u^2 = e^{\ln 2} - 1 = 2 - 1 = 1 : \; u = 1.$

$$I = 6\left[\frac{u^3}{3} + u\right]_1^2$$

$$= 6\left[\frac{8}{3} + 2 - \frac{1}{3} - 1\right]$$

$$= 6\left(\frac{7}{3} + 1\right)$$

$$= \frac{6 \times 10}{3}$$

$$= \underline{\underline{20}}$$

15) C: $y = \dfrac{2x^2 - 1 - 2\ln x^x}{x}$

We note $\ln x^x = x \ln x$,

so $y = \dfrac{2x^2 - 1 - 2\ln x^x}{x}$

$= 2x - \dfrac{1}{x} - \dfrac{2\cancel{x}\ln x}{\cancel{x}}$

$= 2x - \dfrac{1}{x} - 2\ln x$

$\dfrac{dy}{dx} = 2 + \dfrac{1}{x^2} - \dfrac{2}{x}$

$\dfrac{d^2y}{dx^2} = \dfrac{-2}{x^3} + \dfrac{2}{x^2}$ $= 0$ at a point of inflexion, P.

So $\dfrac{2}{x^2} = \dfrac{2}{x^3}$: $\underline{x = 1}$

At $x = 1$, $\dfrac{dy}{dx} = 2 + 1 - 2 = 1$

and $y = 2 - 1 - 2\ln 1 = 1$

So $\underline{\underline{y = x \text{ is a tangent at } P}}$,

as required.

16) A: $(1, 4\sqrt{3})$

B: $(-3+\sqrt{3}, 3)$

gradient of AB is

$$\frac{3-4\sqrt{3}}{(-3+\sqrt{3})-1} = \frac{3-4\sqrt{3}}{-4+\sqrt{3}} = \sqrt{3}\,\frac{-4+\sqrt{3}}{-4+\sqrt{3}} = \sqrt{3}$$

So the line is $y = \sqrt{3}x + c$

where $4\sqrt{3} = \sqrt{3} + c$ (subs. for A)

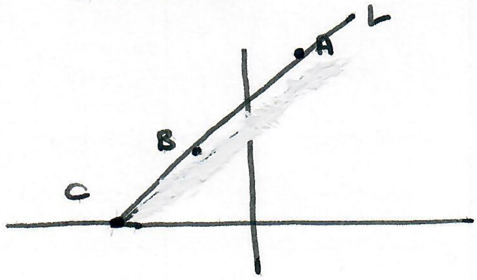
$\therefore$ $\underline{\underline{c = 3\sqrt{3}}}$.

$$y = \sqrt{3}x + 3\sqrt{3} = \sqrt{3}(x+k)$$

So $\underline{\underline{k = 3}}$

At C, $y = 0$ $\quad 0 = \sqrt{3}x + 3\sqrt{3}$

$x = -3$. $\quad$ C is $(-3, 0)$



$$AC^2 = (1-(-3))^2 + (4\sqrt{3}-0)^2$$

$$= 4^2 + 4^2 \cdot 3$$

$$= 4 \cdot 4^2$$

$$= 64$$

So $\underline{\underline{AC = 8}}$

Required angle θ is such that

$$\tan\theta = \sqrt{3}.$$

So $\theta = 60° \left(\frac{\pi}{3}\right)$

17) $P: (20, 60)$

$x = 2at \qquad y = 8at - at^2$

Substitute P:

$20 = 2at \qquad 10 = at \qquad \cancel{t} = \frac{10}{a}$

$60 = 8at - at^2$

$$60 = 8a\frac{10}{a} - a\left(\frac{10}{a}\right)^2$$

$$= 80 - \frac{100}{a}$$

$$20 = \frac{100}{a}$$

$$1 = \frac{5}{a}$$

$$\underline{\underline{a = 5}}$$

So $x = 10t$

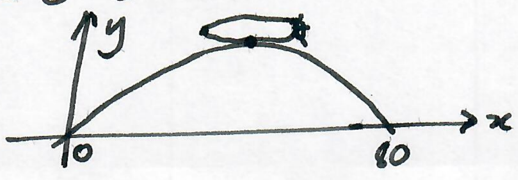
$y = 40t - 5t^2 = 5t(8-t)$

in terms of x:

$t = \frac{x}{10}$ so $y = \frac{40x}{10} - 5\left(\frac{x}{10}\right)^2$

$= 4x - \frac{x^2}{20}.$

$y = 0$ when $t = 0$ or 8, so the curve is a
$x = 0$ or 80
parabola:



c) The point of impact with the airship must be the max. height of the parabola. We could find this by calculating $\frac{dy}{dx}=0$, but in fact since a parabola is symmetric we know it's when $t=4$:

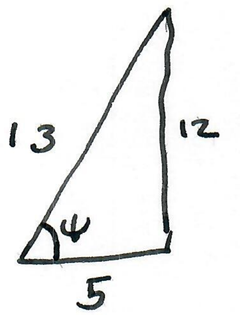
$$x = 2.5.4r = 40\text{m}$$

$$y = 5.4(8-4) = 5.4.4$$

$$= \underline{\underline{80\text{m}}}$$

This is the greatest possible height of the airship.

18) a) $f(\theta) \equiv 5\cos\theta - 12\sin\theta$



$$\frac{f(\theta)}{13} = \frac{5}{13}\cos\theta - \frac{12}{13}\sin\theta$$

$$= \cos\psi\cos\theta - \sin\psi\sin\theta$$

$$= \cos(\psi + \theta)$$

So $f(\theta) = 13\cos(\theta + \psi)$ $\quad$ $\therefore$ 1.176^r

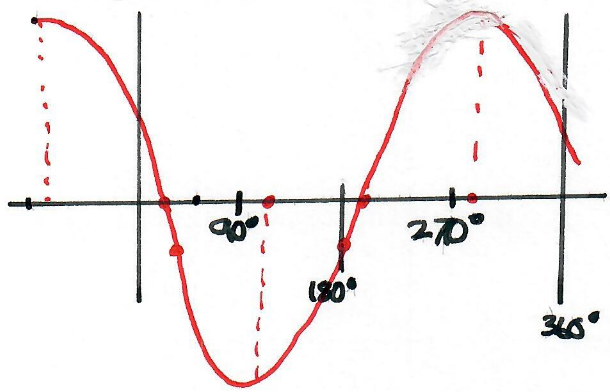
where $\sin\psi = \frac{12}{13}$ $\therefore$ $\psi = \underline{67.380^\circ}$

18) b) Since $f(\theta) = 13\cos(\theta + 67.380°)$

the maximum value of $f(\theta)$ must be 13,

when $\cos(\theta + 67.380°) = 1$.

This could occur when $\theta = -67.380°$, but we require θ to be positive:



So the answer required is $360° - 67.380°$

$= \underline{\underline{292.620°}}$ $= 5.107^c$

c) $P(t) = 20 + 5\cos\left(\frac{4\pi t}{25}\right) - 12\sin\left(\frac{4\pi t}{25}\right)$

(Change $67.380°$ to 1.176^c)

So $P(t) = 20 + 13\cos\left(\frac{4\pi t}{25} + 1.176\right)$

We can use the earlier work to state a max. for this if $\frac{4\pi t}{25}$ gets as high as 5.107,

and happily it does: $\frac{4\pi}{25}(12) = 6.032$.

So we can say the max. $P = 20 + 13 = 33$, and this occurs when $\frac{4\pi t}{25} = 5.107$ or $t = 10.16$ hours (10 hr 9.6 min)

18) a) when $P = 15$:

$$20 + 13\cos\left(\frac{4\pi t}{25} + 1.176\right) = 15$$

$$13\cos\left(\frac{4\pi t}{25} + 1.176\right) = -5$$

$$\cos\left(\frac{4\pi t}{25} + 1.176\right) = \frac{-5}{13} = -0.3846$$

$$\frac{4\pi t}{25} + 1.176 = \cos^{-1}(-0.3846) = 1.9656$$

or -1.9656

i.e. 4.3176

1.9656: $\quad \frac{4\pi t}{25} + 1.176 = 1.9656$: $\quad t = 1.57\text{ hr}$

$= \underline{\underline{1:34}}$

4.3176: $\quad \frac{4\pi t}{25} + 1.176 = 4.3176$: $\quad t = 6.25\text{ hr}$

$= \underline{6:15}$

19) i) $6\tan x = \dfrac{2-3\sec^2 x}{\tan x - 1.}$

so $6\tan^2 x - 6\tan x = 2 - 3\sec^2 x$

$= 2 - 3(1+\tan^2 x)$

$\cos^2 + \sin^2 = 1$

$= 2 - 3 - 3\tan^2 x$

$1 + \tan^2 = \sec^2$

$= -1 - 3\tan^2 x$

$9\tan^2 x - 6\tan x + 1 = 0$

$(3\tan x - 1)(3\tan x - 1) = 0$

$\tan x = \frac{1}{3.}$

so $x = \tan^{-1}\left(\frac{1}{3}\right)$

$= 0.3218$ rad

or $0.3218 + \pi = 3.4633$ rad.

ii) $\cos(3\theta - 60^\circ) = \cos(3\theta + 30^\circ)$

on the face of it this means

$3\theta - 60 = 3\theta + 30$

but that means $0 = 90$
which is impossible

So use the expansions:

$$\cos(3\theta - 60^\circ) = \cos 3\theta \cos 60^\circ + \sin 3\theta \sin 60^\circ$$

$$= \frac{1}{2}\cos 3\theta + \frac{\sqrt{3}}{2}\sin 3\theta \longrightarrow ①$$

$$\cos(3\theta + 30^\circ) = \cos 3\theta \cos 30^\circ - \sin 3\theta \sin 30^\circ$$

$$= \frac{\sqrt{3}}{2}\cos 3\theta - \frac{1}{2}\sin 3\theta \longrightarrow ②$$

we know ① = ②

so removing the $\frac{1}{2}$ factor:

$$\cos 3\theta + \sqrt{3}\sin 3\theta = \sqrt{3}\cos 3\theta - \sin\theta$$

$$\sin 3\theta(\sqrt{3}+1) = \cos 3\theta(\sqrt{3}-1)$$

$$\text{So } \tan 3\theta = \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{(\sqrt{3}-1)(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$$

$$= \frac{3+1-2\sqrt{3}}{3-1} = \frac{4-2\sqrt{3}}{2}$$

$$= 2-\sqrt{3}$$

$$= 15^\circ \text{ or } 195^\circ \text{ or } 375^\circ$$

$$\text{So } \underline{\underline{\theta = 5^\circ \text{ or } 65^\circ \text{ or } 125^\circ}}$$

21) a) C: $y = 3^x + 1 \quad x \in \mathbb{R}$

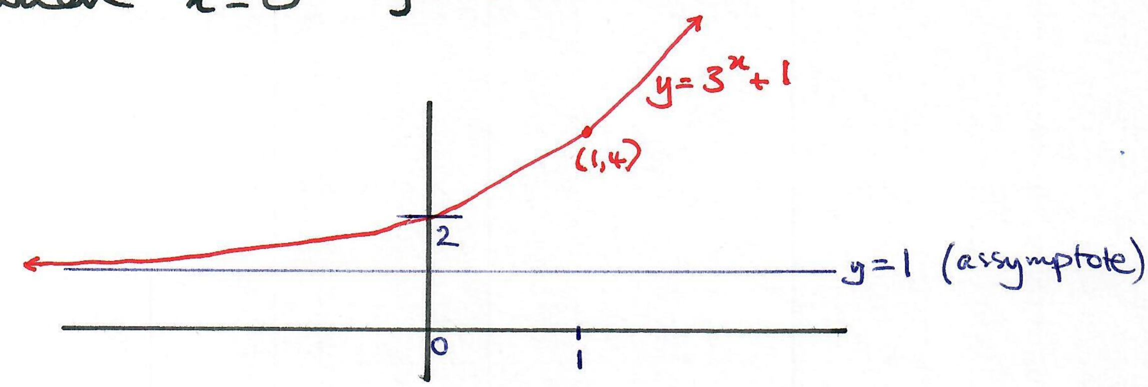
As $x \to \infty$ $3^x \to \infty$, so this curve is 'just one up'

from $y = 3^x$.

As $x \to -\infty$ $3^x \to 0$, so $3^x + 1 \to 1$ from above

When $y = 0$ there is no solution for x

when $x = 0$ $y = 3^0 + 1 = 2$



b) Reflect C in the x axis:

$$y = -(3^x + 1) = -3^x - 1$$

Reflect in the y-axis:

$$y = -3^{-x} - 1$$

(you might express this as

$$3^x y = -1 - 3^x \;)$$

c) First trans^n would be to move the curve to the left by 1

Then move it up by +2

So 1) shift left by 1
2) shift up by 2

which means a translation of $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$

d) This seems puzzling after we've just found 2 translations, but the clue is in the expression : what was $y = 3^x + 1$ is now $y' = 3(3^x + 1)$

So the new curve is actually just an upward stretch by a factor 3.

21) $\dfrac{4t^2}{t-1}$: Divide $4t^2$ by $t-1$

$$4t^2 = 4(t-1)^2 + 8t - 4 \qquad 4(t-1)^2 = 4t^2 - 8t + 4$$

$$= 4(t-1)^2 + 8(t-1) + 4$$

So $4t^2 = 4(t-1)^2 + 8(t-1) + 4.$

$$\frac{4t^2}{t-1} = 4(t-1) + 8 + \frac{4}{t-1}$$

$$= 4t + 4 + \frac{4}{t-1}$$

So $A = 4$
$B = 4$
$C = 4.$

Check: $4t(t-1) + 4(t-1) + 4$
$= 4t^2 - 4t + 4t - 4 + 4$ ✓.

b) $$\int_{16}^{81} \frac{1}{x^{\frac{1}{2}} - x^{\frac{1}{4}}}\,dx.$$

Let $t = x^{\frac{1}{4}}$

then $dt = \frac{1}{4} x^{-\frac{3}{4}}\,dx$

$= \frac{1}{4t^3}\,dx$

$4t^3\,dt = dx$

So the integral is $\displaystyle\int \frac{4t^3}{t^2 - t}\,dt$

Limits: $x = 16 \quad t = 2$
$x = 81 \quad t = 3.$

$$I = \int_2^3 \frac{4t^2}{t-1}\,dt$$

$$= \int_2^3 4t + 4 + \frac{4}{t-1}\,dt$$

$$= \Big[2t^2 + 4t + 4\ln(t-1)\Big]_2^3$$

$$= 18 + 12 + 4\ln(2) - 8 - 8 - \underbrace{\cancel{4\ln(1)}}_{0}$$

$$= \underline{\underline{14 + 4\ln(2)}}$$

22) a) $P = \dfrac{800ke^{0.25t}}{1+ke^{0.25t}}$ $\quad t \geqslant 0$

when $t=0$, $P=175$:

$$175 = \frac{800ke^{0}}{1+ke^{0}} = \frac{800k}{1+k}$$

$$175(1+k) = 800k$$

$$175 + 175k = 800k$$

$$175 = 625k$$

$$k = \frac{175}{625} = \underline{\underline{\frac{7}{25}}}$$

So in general $P = \dfrac{\frac{800\times 7}{25}e^{0.25t}}{1+\frac{7}{25}e^{0.25t}}$

$$= \underline{\underline{\frac{5600e^{0.25t}}{25+7e^{0.25t}}}}$$

i) when $P=560$,

$$560 = \frac{5600e^{0.25t}}{25+7e^{0.25t}}$$

$$14000 + 3920e^{0.25t} = 5600e^{0.25t}$$

$$1680e^{0.25t} = 14000$$

$$e^{0.25t} = 8.333$$

$$0.25t = 2.120$$

$$\underline{\underline{t = 8.481 \text{ years}}}$$

ii) $$P = \frac{5600e^{0.25t}}{25+7e^{0.25t}}$$

As $t \to \infty$, the exponential terms will dominate and 25 become irrelevant –

so in the long term P will $\to \frac{5600}{7}$

i.e. P will stabilise at $\underline{800}$ in the long run.

b) This is a monster job but it becomes simpler if we divide through by $e^{0.25t}$:

$$P = \frac{5600}{25e^{-0.25t}+7}$$

$$\frac{dP}{dt} = \frac{(25e^{-0.25t}+7)0 - 5600(-0.25\times 25e^{-0.25t})}{(\quad)^2}$$

$$= \frac{\frac{5600}{4}\times 25\,e^{-0.25t}}{(\quad)^2}$$

$$= \frac{35000e^{-0.25t}}{(\quad)^2}$$

Also $$\frac{P(800-P)}{3200} = \frac{1}{3200}\times\frac{5600}{(\quad)}\times\left(800-\frac{5600}{(\quad)}\right)$$

$$= \frac{1}{3200}\times\frac{5600}{(\quad)}\times\left(\frac{800(\quad)-5600}{(\quad)}\right)$$

$$= \frac{7}{4(\quad)^2}(20000e^{-0.25t}+\cancel{5600}-\cancel{5600})$$

$$= \frac{35000}{\quad}e^{-0.25t}$$ as required.