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1) $0.\dot{2}\dot{5} = 0.25252525....$

$\frac{2}{5}$ of this $= 0.10101010....$

$= 0.\dot{1}\dot{0}$ (E)

2) twip $= 18 \times 10^{-6}$ m

league $= 48 \times 10^{2}$ m.

league/twip $= \frac{48}{18} \times 10^{2} \times 10^{6}$

$\simeq 2.7 \times 10^{8}$

$= 270,000,000$ (A)

3) Sum of faces on a standard die = 21

So total = 42.

So missing amount is 9.

Opp. faces sum to 7.

So $2x + (7 - x) = 9$

$x + 7 = 9$

$x = 2.$

Check: $2 + 2 + 5 = 9$ ✓.

$+ 33 = 42$ ✓

(B)

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4) $x + 7x + x^2 = 180$

$x^2 + 8x - 180 = 0$

$(x - 10)(x + 18) = 0$

$x = 10.$

Angles are $10^\circ, 70^\circ, \underline{100^\circ}$ (C)

5) $4^5 \times 5^4 = 4 \times (20)^4$

$20^4 =$ ~~400~~ ~~8000~~ 160000

$4 \times 20^4 = 640000$

– 6 digits (B)

6) Min. of 3 faces at each point (octahedral pyramid) so 16 edges. (D)

7) $3^8 - 1 = (3^4 - 1)(3^4 + 1) = 80 \times 82$

$= \cdots \times 41.$

(A)

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8) x is actually irrelevant...
it's a question of which 4 add up to 44. Which is $4+8+12+20$
So $16x$ has been removed.

(D)

9) Puzzling, but it must be even (so 8....8) and the digits must add to a multiple of ~~9~~ in fact 18, since it's palindromic.
So try the 2nd digit as 9 (as big as possible) and we get:

891198 (which is 18×49511)
so the 100s digit is 1. (E)

10) $2024 = 2 \times 2 \times 2 \times 11 \times 23$

Multiples of 23: 23 46 [illegible] 92
11 : 11 22 [illegible] 44 [illegible] 88
8 : [illegible] [illegible] [illegible]
4: [illegible]
2: [illegible] So 7: (D)

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11) A: $n+1$ ✗ $n=2$

B: $n(n+1)+1$ ✗ " $n=2$

C: $n(n+1)(n+2)+1$ ✗ $n=1$

D: $n(n+1)(n+2)(n+3)+1$ $n=1$ ✓ $n=2$ ✓

E: $n(n+1)(n+2)(n+3)(n+4)+1$ $n=1$ ✓ $n=2$ ✗.

D also 'looks' right. Expanding...

$n(n+1)(n^2+5n+6)+1$

$n(n^3+5n^2+6n+n^2+5n+6)+1$

$n^4+5n^3+6n^2+n^3+5n^2+6n+1$

$n^4+6n^3+11n^2+6n+1$

which also looks like a square...

$(n^2+3n+1)(n^2+3n+1)$ ✓ **D**

12) All the 1st digits must be even:

	23		29
41	43	47	
61		67	
	83		89

So we must pick 1 from each row and 1 from each column, giving a total $(20+40+60+80)+(1+3+7+9)$
$= 200+20 = 220$. **A**

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13)

$$
\begin{array}{ccccccc}
 & & & 2024 & & & \\
 & & a & & b & & \\
 & c & & z & & d & \\
1 & & e & & f & & 10
\end{array}
$$

$$
\begin{aligned}
2024 &= a + b \\
&= c + 2z + d \\
&= 1 + e + 2(e+f) + f + 10 \\
&= 1 + 3e + 3f + 10 \\
&= 3e + 3f + 11.
\end{aligned}
$$

$2013 = 3(e+f) = 3z.$

$z = 2013/3 = 671$ **C**

14) $\{P\ Q\ R\ S\ T\} = \{1\ 2\ 3\ 4\ 5\}$

PRT, QRS are both primes...
so between 100 and 599.

But no '100s' number works...

~~127~~ ~~129~~ ~~131~~ ~~133~~ ~~137~~ ~~(143)~~ ~~151~~ ~~153~~

UKMT SMC Challenge 2024 (6)

And 143 would require ~~248~~ or ~~542~~.

Also note T and S can't be 2, 4 or 5.

Primes in the 200s:

~~2 22 23~~ 251 → ~~453~~

253 → ~~451~~

÷ by ~~2 3 5 7~~ 11 ✓

300s:

~~31~~ ~~32~~ ~~33~~ ~~341~~ ~~35~~

400s: (getting desperate)

~~413~~ 421 → 523 ✓ So R is 2.

✓ ✓

~~2~~ ~~3~~ ~~7~~ ~~11~~ ~~13~~ ~~17~~ ~~19~~

[B]

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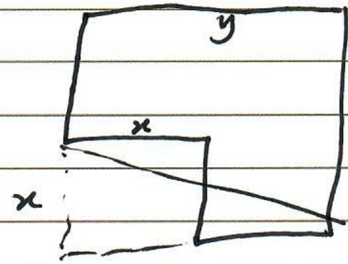
15)



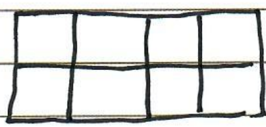
$y^2 - x^2 = 62$
$y^2 + x^2 = 100$

$2y^2 = 162$
$y^2 = 81$
$y = 9$

$x^2 = 19$
$x = \sqrt{19}$

C

16)



3 red doors
≥ 1 red in each row.
≥ 2 red corners

R 1 2 3
R 4 5 6

6 options

. . . R
. . . R

6

R 1 2 3
3 4
4 3 4 R

4

– 1 2 R
R 3 4 –

4

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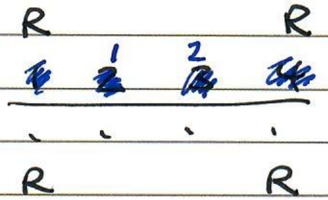


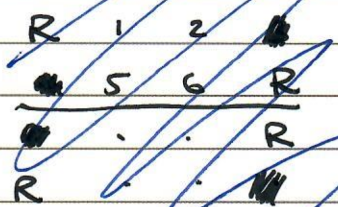
2

2

24 B

So I think it's 6+6+6+6+4+4 = 32.

but also ...



4

4

which makes 40.

but some of these duplicate what we've already got.

I can't think of another 16, so I'll say it's 40. D B

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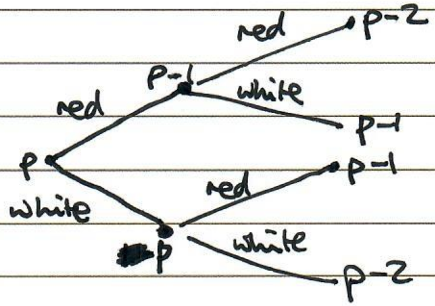
17)



$p =$ num of red orig. in bag $=$ 1, 2 or 3.

(assume not 4 or 0)

Prob that both are red $= \frac{p}{4} \cdot \frac{p-1}{3} = \frac{1}{2}$

$p(p-1) = 6$

$p^2 - p - 6 = 0$

$(p-3)(p+2) = 0$

$p = 3$ (or -2)

So p(both are white) $=$ 0

Put another way: R R R W $\quad p_1(R) = \frac{3}{4}$ $\quad p_2(R) = \frac{2}{3}$

$p_1 p_2 = \frac{1}{2}$ ✓. So $P(WW) = 0$.

E

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18) Area of inner circle (πr^2)

$= \pi$

Area of outer circle

$= \pi(1+x)^2$

Area of $\frac{1}{4}$ inner circle $= \frac{\pi}{4}$.

So $\pi(1+x)^2 = 14 \cdot \frac{\pi}{4} = \frac{7}{2}$

$(1+x)^2 = \frac{7}{2}$

$1 + 2x + x^2 = \frac{7}{2}$

$x^2 + 2x - \frac{5}{2} = 0$

$2x^2 + 4x - 5 = 0$

$x = \frac{-4 \pm \sqrt{16+40}}{4} = -1 \pm \frac{\sqrt{14}}{2}$

$\therefore \frac{\sqrt{14}}{2} - 1$

A C

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19)

P	4	1/3	∴ A is false
Q	11	2/9 4/7 5/6	∴ B is false.
R	16	4/12 5/11 6/10 7/9	not 8
S	19	7/12 8/11 9/10	
T	20	8/12 9/11	not 10.

if T = 8/12 then S = 9/10 then R = 5/11 then Q = 4/7

if T = 9/11 then S = 7/12 then R = 6/10 then Q = imp.

So T = 8/12   E is false
S = 9/10   D is false
R = 5/11   C is true.

C

20) $\frac{1}{x} + \frac{1}{y} = \frac{1}{20}$

D

$\frac{1}{20} - \frac{1}{420} = \frac{21}{420} - \frac{1}{420} = \frac{20}{420} = \frac{1}{21}.$

but $\frac{1}{20} - \frac{1}{480}$ doesn't work like this.

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21).

[1] 2	9	[2] 7
5	▨	4
[3] 3	6	1

2 down must be a multiple of $13 \times 19 = 247$.

$247 \times 1 = 247$
$\times 2 =$ ~~494~~
$\times 3 = 741$
$\times 4 =$ ~~988~~

3 across is a square, which can't end in 7.
So 2 down is 247.

3-digit squares ending in 1:

$11^2 =$ ~~121~~
$19^2 = 361$
$21^2 =$ ~~441~~
$29^2 =$ ~~841~~

So 3 across is 361

A multiple of 11 ending in 3 must be 3×11:

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13 × 11 = ~~143~~
23 × 11 = 253
33 × 11 = ~~363~~
43 × 11 = ~~473~~
53 × 11 = 583
63 × 11 = ~~693~~
73 × 11 = ~~803~~
83 × 11 = ~~913~~

So 1 down is 253 or 583.

1 across is div. by 9, so
253 would mean 1 across is 297
583 - - - - - - ~~567~~

So 1 down is 253
1 across is 297.

Missing digit is 8 **B**

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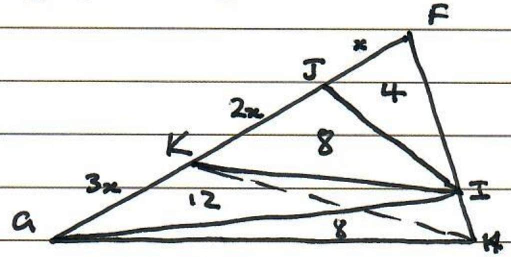
22)



Lengths along GF are in prop^n 3:2:1

$\triangle$ ~~G~~KH = $\frac{1}{2}$ $\triangle$ GFH = $\frac{1}{2}(12+8+8+4)$

$= \frac{1}{2}(32) = 16.$

$\triangle$ ~~EG~~ FKI = 12.

so $\triangle$ KIH = 16 − 12 = 4. [A]

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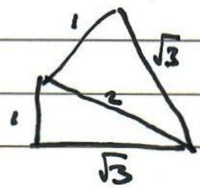
23) Kite area $= \sqrt{3}$.

which means



Area $= \frac{1 \cdot \sqrt{3}}{2} \times 2 = \sqrt{3}$.

So the edges are $1, \sqrt{3}, 2$.

On the hat tile,
perimeter $= 8 \times 1 + 6 \times \sqrt{3}$
$= 8 + 6\sqrt{3}$

E

24) $f(x) + f\left(\frac{1}{1-x}\right) = 24x$

except $x = 0, 1$. Find $f(3)$.
There's not much we can do except try:
$f(3) + f\left(-\frac{1}{2}\right) = 72$.

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And keep going:

$$f\left(-\frac{1}{2}\right) + f\left(\frac{1}{1-\left(-\frac{1}{2}\right)}\right) = 24 \cdot -\frac{1}{2} = -12$$

$$f\left(-\frac{1}{2}\right) + f\left(\frac{2}{3}\right) = -12$$

It's not looking good, but...

$$f\left(\frac{2}{3}\right) + f\left(\frac{1}{1-\frac{2}{3}}\right)$$

evidently $x \to \frac{1}{1-x}$ cycles $\left\{3, -\frac{1}{2}, \frac{2}{3}\right\}$

$$= f\left(\frac{2}{3}\right) + f(3) = 24 \cdot \frac{2}{3} = 16.$$

So... let $f(3) = a$

$f\left(-\frac{1}{2}\right) = b$

$f\left(\frac{2}{3}\right) = c.$

Then

$a + b = 72$ ①

$b + c = -12$ ②

$c + a = 16.$ ③

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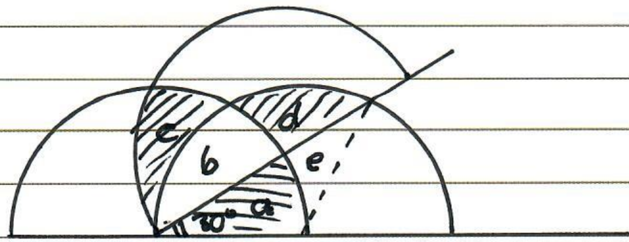
③ − ②: $a - b = 16 - (-12) = 28$

$a + b = 72$

$2a = 100$

$a = 50$ **E**

25)



This one looks tough – it's tempting to label all the regions and even resort to areas of segments/sectors/lines ... but actually it's quite simple in terms of overlapping areas, and with a couple of extra lines.

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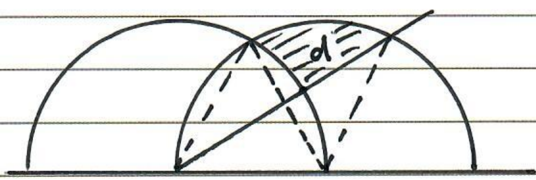
a) The 30° angle tells us a is $\frac{1}{6}$ of a semicircle $= \frac{1}{6} \times 24 = 4$.

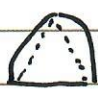
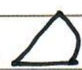
b) The  area ab is congruent to bc, so

$$a + b = b + c$$

$$\text{So } c = a = 4.$$

c) To find d:



we use:  =  +  − 

= 8 − 

and  =  (same base + height)

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And  = 8.

So d = 

= 8 − (8 − △) − △ + 4

= ~~8~~ − ~~8~~ + ~~△~~ − ~~△~~ + 4

= 4.

So a + c + d = 4 + 4 + 4 = 12 **[A]**