1) Not quite sure whether this wants 'separated fractions', or just a simplified form: the published answers indicate the latter.

$$\frac{x}{(x+1)(x+3)} + \frac{x+12}{(x^2-9)}$$

$$= \frac{x}{(x+1)(x+3)} + \frac{x+12}{(x-3)(x+3)}$$

$$= \frac{1}{(x+3)}\left[\frac{x}{x+1} + \frac{x+12}{x-3}\right]$$

$$= \frac{1}{(x+3)}\left[\frac{x(x-3)+(x+12)(x+1)}{(x+1)(x-3)}\right]$$

$$= \frac{1}{(x+3)}\left[\frac{x^2-3x+x^2+13x+12}{(x+1)(x-3)}\right]$$

$$= \frac{1}{(x+3)}\frac{(2x^2+10x+12)}{(x+1)(x-3)}$$

$$= \frac{2}{(x+3)}\frac{(x^2+5x+6)}{(x+1)(x-3)}$$

$$= \frac{2}{\cancel{(x+3)}}\frac{\cancel{(x+3)}(x+2)}{(x+1)(x-3)} = \frac{2(x+2)}{(x+1)(x-3)}$$

2) Binomial expansion:

$$(1+px)^n = 1 + npx + \frac{n(n-1)}{2}(px)^2 + \cdots$$

Equating with the given expression,

Coeffs of $x$ : $np = -18$ —— ①

" " $x^2$ : $\frac{n(n-1)}{2}p^2 = 36p^2$ —— ②

from ②:

$$\frac{n(n-1)}{2} = 36$$

$$n(n-1) = 72$$

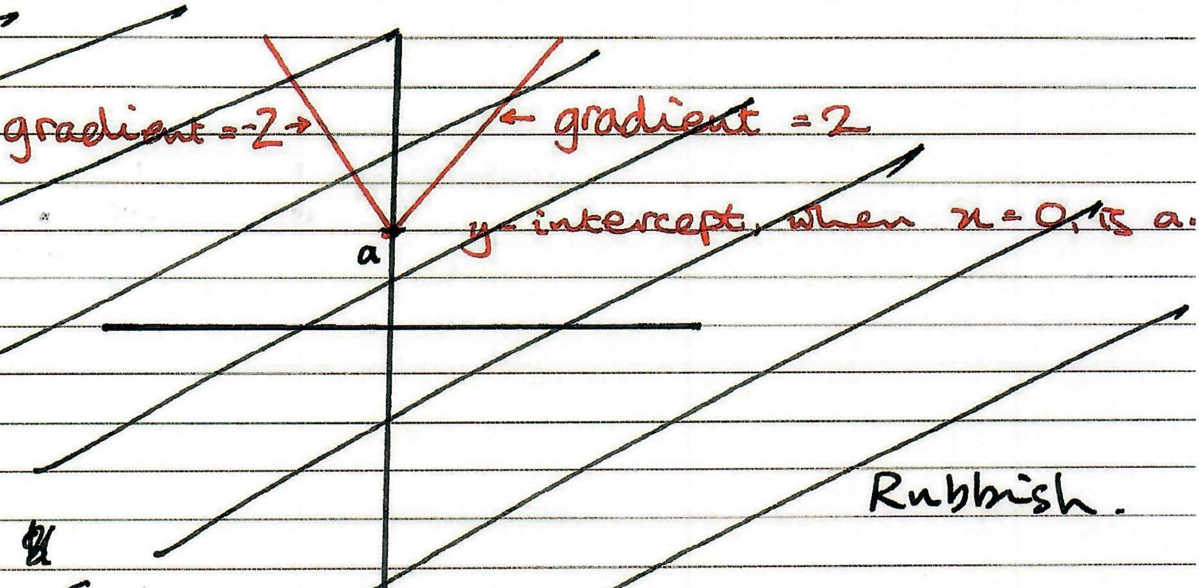
$$\underline{\underline{n = 9}}.$$

from ①:

$$p = -\frac{18}{9} = -2$$

So $\underline{\underline{n = 9 \text{ and } p = -2.}}$

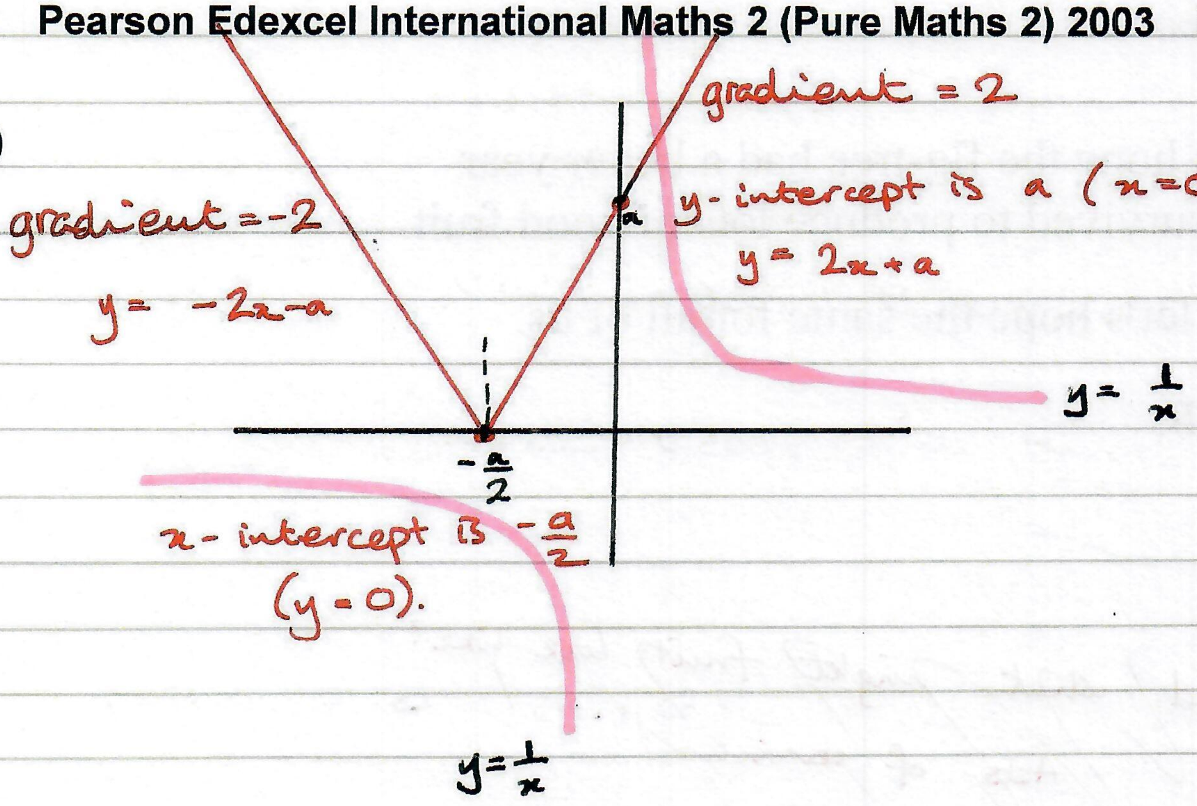
~~3)~~




Rubbish.

a) $2\sqrt{2}$

3)




c) Left-hand side of the V is entirely in the top-l quadrant, where $y = \frac{1}{x}$ doesn't go.

Right-hand side of the V meets $y = \frac{1}{x}$ at only 1 point.

To do it analytically:

LHS : $-2x - a = \frac{1}{x}$ : $-2x^2 - ax = 1$

$$2x^2 + ax + 1 = 0$$

$$x = \frac{-a \pm \sqrt{a^2 - 8}}{4}$$

This has solutions, but only for y<0 ... where the V curv doesn't go

RHS: $2x + a = \frac{1}{x}$

$2x^2 + ax - 1 = 0$

$x = \frac{-a \pm \sqrt{a^2 - 8}}{4}$ : $\mathbb{R}$ when $a > 2\sqrt{2}$ and then there's o one > value of $x$.

(actually this gets very tedious … the published answer doesn't need all this).

d) $x|2x+1| - 1 = 0$ ——— ①

This is much the same equation:

$x(2x+1) - 1 = 0$

$2x^2 + x - 1 = 0$

$x = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm 3}{4} = -1$ (discar since ① not be tru

or $\underline{\frac{1}{2}}$

4) Volume of revolution about the $x$-axis is:

$$\int_1^4 \pi y^2 \, dx$$

$$= \pi \int_1^4 \left(1 + \frac{1}{2\sqrt{x}}\right)^2 dx$$

$$= \pi \int_1^4 \left(1 + \frac{2}{2\sqrt{x}} + \frac{1}{4x}\right) dx$$

$$= \pi \int_1^4 1 + x^{-\frac{1}{2}} + \frac{1}{4}x^{-1} \, dx$$

$$= \pi \left[ x + \frac{2}{1}x^{\frac{1}{2}} + \frac{1}{4}\ln x \right]_1^4$$

$$= \pi \left[ (4-1) + 2\left(4^{\frac{1}{2}} - 1^{\frac{1}{2}}\right) + \frac{1}{4}(\ln 4 - \ln 1) \right]$$

$$\therefore = \pi \left[ 3 + 2 + \underset{\frac{1}{2}\ln 2}{\cancel{\tfrac{1}{4}\ln 4}} \right] = \underline{\underline{\pi\left(5 + \frac{1}{2}\ln 2\right)}}$$

as require

5) a)

| $x$ | 0 | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|
| $y$ | 0 | 6.13 | 7.80 | 7.60 | 6.13 | 0 |

$$y = 8\sqrt{\sin \frac{\pi x}{10}}$$

b) Using the trapezium rule,

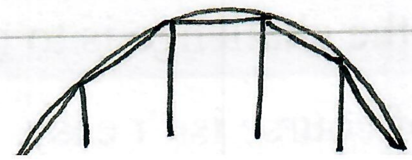
$$\int_0^{10} y\,dx \approx \frac{1}{2} \times 2 \{ (0+0) + 2(6.13 + 7.80 + 7.80 + 6.13)$$

$$= \{ 4 \times 13.93 \} = 54.96$$

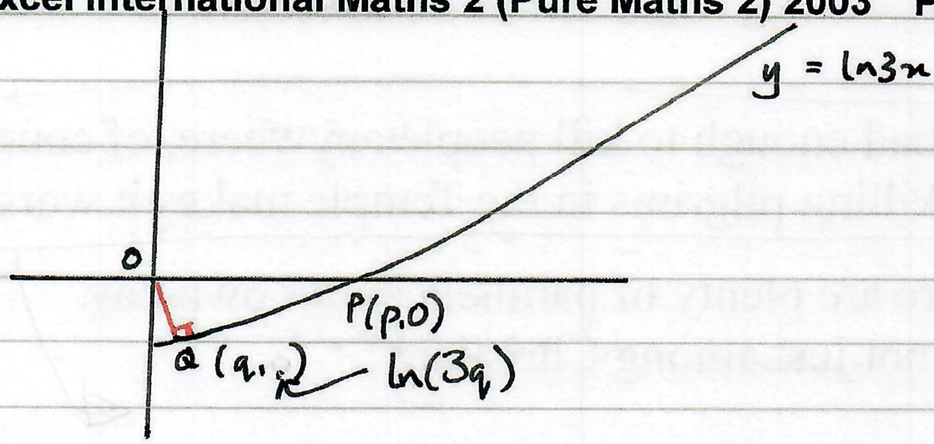
$$= \underline{\underline{55.72\ m^2}}$$

c) Concrete surround $= 10 \times 14. - 55.72 = \underline{\underline{84.}}$

d) The answer in (c) over-estimates because the answer in (b) under-estimates... which is because each trapezium falls short of the curve, which is always above the edges of the trapezium:



6)




At P, $y = 0 = \ln 3x$ so $3x = 1$: $x = \frac{1}{3}$.

On the curve, $\frac{dy}{dx} = \frac{3}{3x} = \frac{1}{x}$

So the gradient at Q is $\frac{1}{q}$.

So the gradient of the normal (= negative recipr
is $-q$.

But the gradient is also

$$\frac{\ln(3q) - 0}{q - 0} = \frac{\ln(3q)}{q}$$

So $\frac{\ln(3q)}{q} = -q$: $\ln(3q) = -q^2$: $q^2 + \ln(3q) = 0$

So $q$ is a solution of $x^2 + \ln(3x) = 0$ as required

c) $x^2 + \ln 3x = 0$

$\ln 3x = -x^2$

exp: $3x = e^{-x^2}$

$$x = \frac{1}{3}e^{-x^2}$$

d) $x_{n+1} = \frac{1}{3}e^{-x_n^2}$

| $n$ | $x_n$ | |
|---|---|---|
| 0 | $\frac{1}{3}$ | 0.3333 |
| 1 | | 0.2983 |
| 2 | | 0.3050 |
| 3 | | 0.3037 |
| 4 | | 0.3040 |

So to 3dp, $q = 0.304$.

7) a) Use the identity

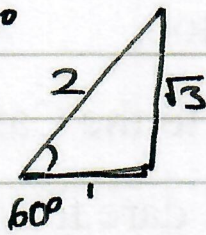
$$R\sin(x+\alpha) = R(\sin x\cos\alpha + \cos x\sin$$

$$\sin x + \sqrt{3}\cos x = R\sin x\cos\alpha + R\cos x\sin$$

So $R\cos\alpha = 1$

and $R\sin\alpha = \sqrt{3}$.

So $\tan\alpha = \sqrt{3}$ : $\alpha = 60°$




$\cos\alpha = \frac{1}{2}$

So $R \times \frac{1}{2} = 1$ : $R = 2$.

So the given expression

$$\sin x + \sqrt{3}\cos x = \underline{\underline{2\sin(x+60°)}}. \quad \text{—} \ (1$$

b) $\sec x + \sqrt{3}\,\text{cosec}\, x = 4$ —— (3)

multiply throughout by $\sin x\cos x$:-

$\cancel{\sec x} \sin x \cancel{\cos x} + \sqrt{3}\, \cancel{\operatorname{cosec} x}\, \cancel{\sin x} \cos x$

$= 4 \sin x \cos x$

$= 2 \sin 2x$

So $\sin x + \sqrt{3} \cos x = 2 \sin 2x$ — as required.

c) Substituting ① into ②:

$$2 \sin (x + 60°) = 2 \sin 2x$$

So $\sin 2x - \sin (x + 60°) = 0$ as require
with $x$ satisfying ③ as give

d) Applying the given identity:

$\sin 2x - \sin (x + 60°) = 0$

$$= 2 \cos \left( \frac{3x + 60°}{2} \right) \sin \left( \frac{x - 60°}{2} \right)$$

So Solutions as required are given by

$$\cos\left(\frac{3x+60^\circ}{2}\right) = 0$$

$$\frac{3x+60^\circ}{2} = 90^\circ,\ 270^\circ,\ 450^\circ \ \ldots\ldots$$

$$3x+60^\circ = 180^\circ,\ 540^\circ,\ 900^\circ \ \ldots\ldots$$

$$3x = 120^\circ,\ 480^\circ,\ \ldots$$

$$\underline{\underline{x = 40^\circ,\ 160^\circ.}}$$

$$\sin\left(\frac{x-60^\circ}{2}\right) = 0$$

$$\frac{x-60^\circ}{2} = 0^\circ,\ 180^\circ,\ \ldots$$

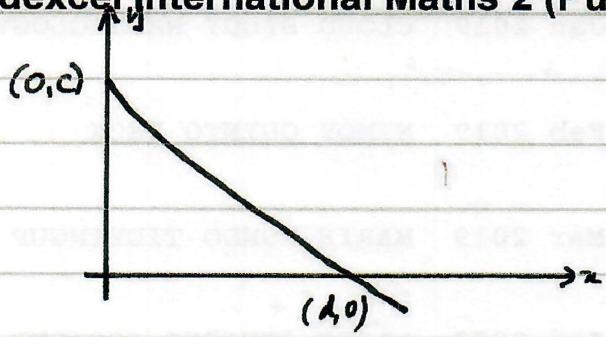
$$x-60^\circ = 0^\circ$$

$$\underline{\underline{x = 60^\circ}}$$

So the solutions in the required range are $x = 40^\circ$

It

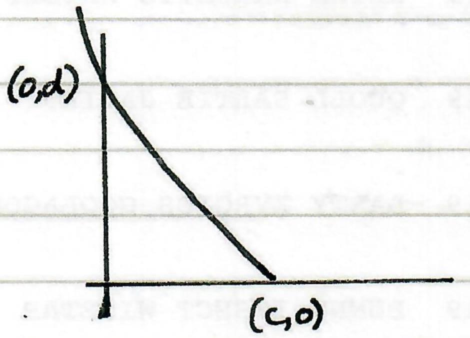
8)




$c = f(0)$

$0 = f(d)$

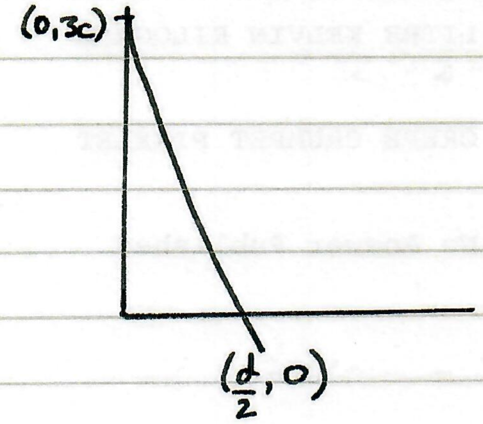
a)




$0 = f^{-1}(c)$

$d = f^{-1}(0)$

b)




$y = 3f(0) = 3c$

$y = 3f\left(\frac{2d}{2}\right) = 0$

c) $f(x) = 3(2^{-x}) - 1$ $x \in \mathbb{R}$ etc...
$x \geqslant 0$

$C = f(0) = 3(2^{-0}) - 1 = 3 \times 1 - 1 = \underline{\underline{2}}.$

The range of $f$ is the set of values for all possible $x$, which is:

$x = 0$: $f(x) = 2$

$x \to \infty$ $f(x) = \dfrac{3}{2^x} - 1$

$2^x$ very large, $\dfrac{3}{2^x}$ very small

$\to -1.$

$f$ is continuous and uniquely valued:

$y = 3(2^{-x}) - 1$

$\dfrac{y+1}{3} = 2^{-x} = \dfrac{1}{2^x}$

$x = \log_2\left(\dfrac{3}{1+y}\right)$

So range $f = (-1, 2]$

or $-1 < f(x) \leq 2.$

d) As in c, $x = \log_2\left(\frac{3}{1+y}\right)$

so $d = \log_2\left(\frac{3}{1}\right) = \log_2(3)$

$= \frac{\ln(3)}{\ln(2)}$

$= \underline{\underline{1.585.}}$

e) $g: x \mapsto \log_2 x$

$g(x) = \log_2 x$

$fg(x) = 3\left(2^{-\log_2 x}\right) - 1$

$= \frac{3}{2^{\log_2 x}} - 1 = \underline{\underline{\frac{3}{x} - 1}}$