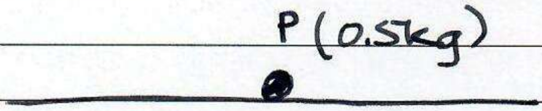
1.)



a) P is at rest, so the grav. force 0.5×9.8 is balanced by the normal reaction-

which is $\underline{\underline{4.9N.}}$

b) frictional force at max is $4.9N \times \frac{2}{7}$

$= \underline{\underline{1.4N}}$

2.) a) speed increases from 0 to $10ms^{-1}$ in 4 sec

So acceleration $= \frac{10}{4} ms^{-2} = 2.5ms^{-2}$

b) Distance is the area under the graph ($\int v\,dt$):

1st 4 s	$\frac{10 \times 4}{2}$	= 20 m
4s - 18s	10×14	= 140m

stupid!

Total: $\underline{\underline{160m}}$

Athlete covers the last 40m in 6 s, starting at speed 10.

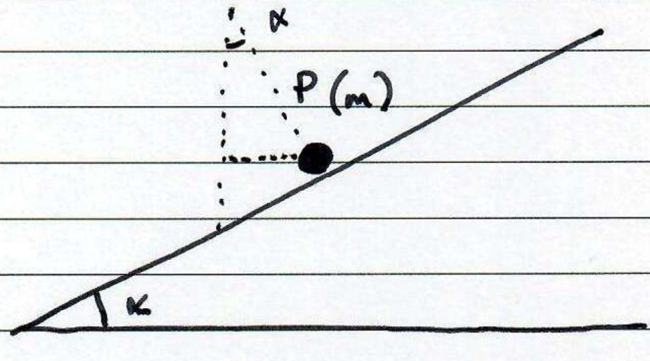
So (area of a trapezium):

$$\frac{6\times(10+u)}{2} = 40$$

$$60 + 6u = 80$$

$$u = \frac{20}{6} = 3\tfrac{1}{3}\ \mathrm{ms}^{-1}$$

3)



$\tan\alpha = \frac{5}{12}$

$\mu < \frac{5}{12}$

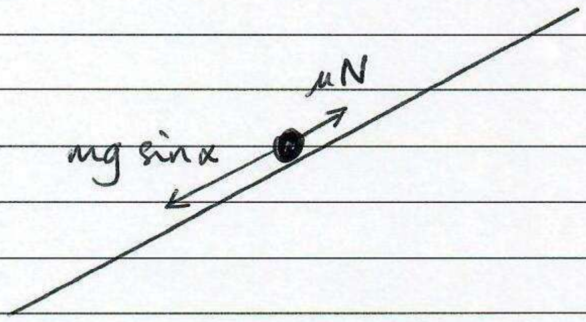
Weight (force) of $P = mg$: resolve this to

Normal reaction from plane =

$$mg\cos\alpha = mg\cdot\frac{12}{13}$$

$$= \frac{12}{13}mg.$$

In the direction of the plane:



R = Resultant force on the particle is

$$mg \sin \alpha - \mu N = mg(\sin \alpha - \mu \cos \alpha)$$

$$= \frac{mg(5 - 12\mu)}{13}$$

So acceleration is $\frac{R}{m} = \frac{1}{13} g(5 - 12\mu)$

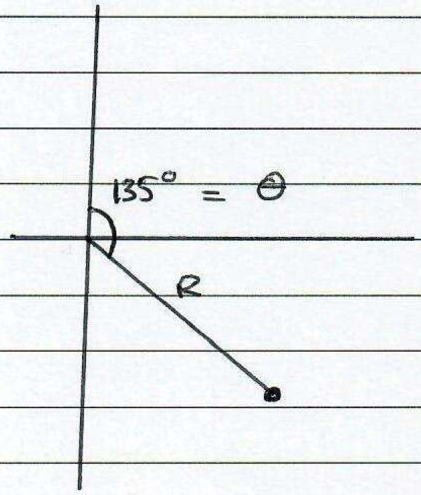
d) If $\mu \geqslant \frac{5}{12}$ the particle wouldn't move.

4)

$\uparrow \underline{j} \; \rightarrow \underline{i}$

$$\underline{r} = ct^{\frac{1}{2}}\,\underline{i} - \frac{3}{8}t^2\,\underline{j}$$

a)



At this point

$$\underline{r} = \cancel{\cos\theta}\; R\sin\theta\,\underline{i} \quad R + \cos\theta\,\underline{j}$$

and θ is $135°$:

$$c\sqrt{4}\,\underline{i} - \frac{3}{8}16\,\underline{j} = \frac{R}{\sqrt{2}}\,\underline{i} - \frac{R}{\sqrt{2}}\,j$$

Resolving:

$$\left.\begin{array}{r} 2c = R/\sqrt{2} \\ -6 = R/\sqrt{2} \end{array}\right] \quad \underline{\underline{c = -3.}}$$

as required

Velocity of P is $\underline{v} = \frac{d\underline{r}}{dt}$

Speed of P is $\left|\frac{d\underline{r}}{dt}\right|$

$$\underline{v} = \frac{1}{2} \cdot 3\, t^{-\frac{1}{2}}\, \underline{i} - 2 \cdot \frac{3}{8}\, t\, \underline{j}$$

$$= \frac{3}{2} \frac{1}{\sqrt{t}}\, \underline{i} - \frac{3}{4}\, t\, \underline{j}$$

when $t = 4$:

$$= \frac{3}{4}\underline{i} - 3\underline{j}$$

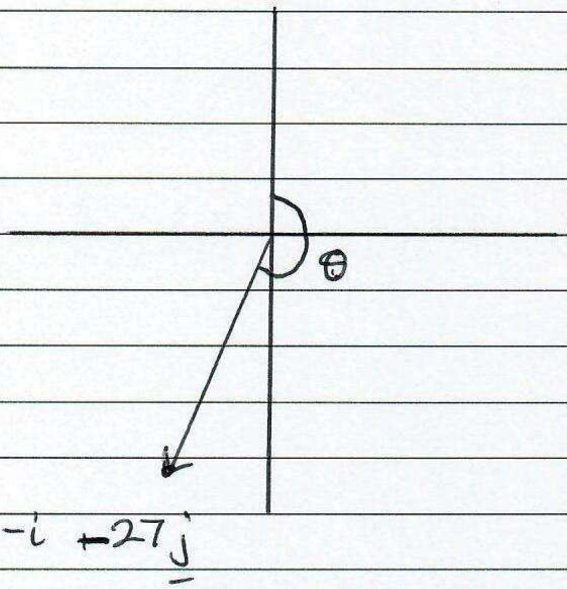
So speed of P $= \sqrt{\left(\frac{3}{4}\right)^2 + 3^2}$

$$= \sqrt{\frac{17}{16} \times 9}$$

$$= \frac{3}{4}\sqrt{17} = 3.0923 \text{ ms}^{-1}.$$

d) Acceleration $\underline{a}$ is $\frac{d\underline{v}}{dt} = -\frac{1}{2}\frac{3}{2}\, t^{-\frac{3}{2}}\, \underline{i} - \frac{3}{4}\underline{j}$

$$= -\frac{3}{4}\, t^{-\frac{3}{2}}\, \underline{i} - \frac{3}{4}\underline{j}$$



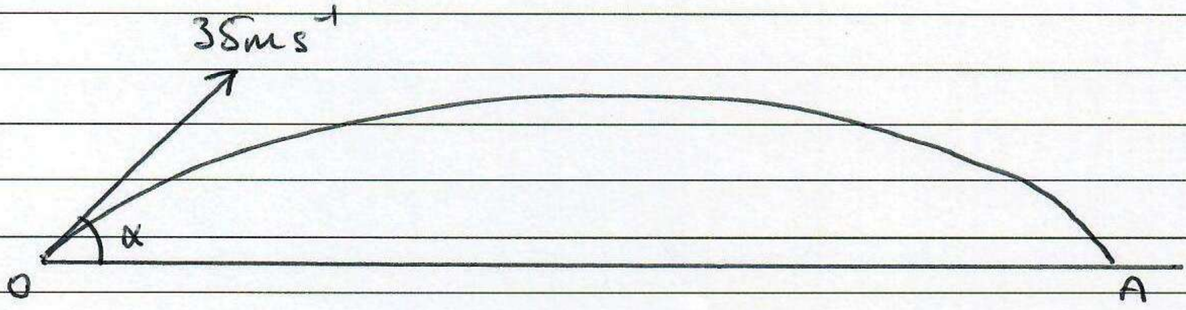
So $\quad -\frac{3}{4}T^{-\frac{3}{2}} = R\cos\theta$

$\quad -\frac{3}{4} = R\sin\theta$

and $\quad \frac{\sin\theta}{\cos\theta} = \frac{1}{27} = T^{-\frac{3}{2}}$

So $\quad T^{\frac{3}{2}} = 27: \quad \underline{\underline{T = 9 \ (s)}}$

5)



$\tan \alpha = \frac{3}{4}$ (Pythagorean)

using SUVAT in the vertical direction
we need s, u, a, t:

$$s = ut + \frac{at^2}{2}$$

or $$y = 35 \sin\alpha \, t - \frac{gt^2}{2}$$

$$y = 21t - \frac{gt^2}{2}$$

but also, $x = 35\cos\alpha \, t = 35.\frac{4}{5}t$

so $t = \frac{5\cancel{8}}{35.4\cancel{4}}x.$

So $y = \frac{21.5}{35.4}\cancel{t}x - \frac{g}{2}\left(\frac{5}{\cancel{4}\,35.4}\right)^2 x^2$

$= \cancel{\frac{168}{4}x - \frac{5\times 9.8}{8}x^2}$

$= \frac{3}{4}x - \frac{9.8}{98}x^2 = \frac{3}{4} - \frac{1}{160}x^2.$

b) At O and A,

$$y = 0 = \frac{3}{4}x - \frac{1}{160}x^2 = x\left(\frac{3}{4} - \frac{1}{160}x\right)$$

$x = 0$ is one solution (O): A is given by the other factor: $\frac{3}{4} - \frac{1}{160}x = 0$

$$\frac{1}{160}x = \frac{3}{4} \qquad x = 160 \times \frac{3}{4} = \underline{\underline{120\text{m}}}.$$

c) H is given by $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{3}{4} - 2\frac{1}{160}x = \frac{3}{4} - \frac{x}{80}$$

So $\frac{x}{80} = \frac{3}{4} \qquad x = \frac{240}{4} = 60$

So $y = H = \frac{3}{4}.60 - \frac{1}{160} \times 3600$

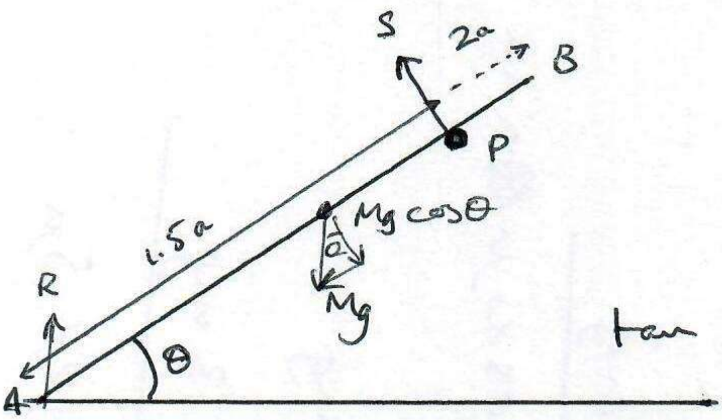
$$= 45 - \frac{90}{4} = 45 - 22.5$$

$$= \underline{\underline{22.5\text{ m}}}.$$

d) Air resistance would (slow the particle down) add a downward force – so K would be lower than H.

e) The given answer says $g = 9.8$ would be wrong – but pres. you've already taken that into account in the new model?

6)



$\tan\theta = \frac{4}{3}$ (oops!)

$\left(\sin\theta = \frac{4}{5};\ \cos\theta = \frac{3}{5}\right)$

a) Moments about A:

$$1.5aS = 1a\,Mg\cos\theta = \frac{3}{5}Mg\,a$$

So $S = \frac{2}{5}Mg$ as required.

b) Forces acting on the rod:

horizontally: $F = S\sin\theta = \frac{2}{5}\cdot\frac{4}{5}Mg = \frac{8}{25}Mg$

and $F = \mu R$

vertically:

$R + S\cos\theta = Mg$ so $R = Mg - \frac{3}{5}\frac{2}{5}Mg$

$= Mg\left(1 - \frac{6}{25}\right) = Mg\left(\frac{19}{25}\right)$

So $F = \frac{8}{25} Mg = \mu R = \mu Mg \left(\frac{19}{25}\right)$

So $\mu = \frac{8}{19} = 0.42105.$