1) a) X is described by a Binomial Distribution denoted as $B(10, \frac{1}{6})$

i) $P(X=3)$ is the $\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^7$ term in the expansion of $\left(\frac{1}{6}+\frac{5}{6}\right)^{10}$, given by

$${}_{10}C_3 \quad \left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^7$$

$$= \frac{10 \times \cancel{9}^{3} \times \cancel{8}^{4}}{\cancel{3 \times 2 \times 1}} \left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^7$$

$$= \underline{\underline{0.1550}}$$

Similarly

$$P(X=2) = \frac{\cancel{10}^{5} \times 9}{\cancel{2}} \left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^8 = 0.2907$$

$$P(X=1) = 10\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^9 = 0.3230$$

$$P(X=0) = \left(\frac{5}{6}\right)^{10} = \underline{0.1615}$$

$$\text{Total} = \underline{\underline{0.7752}}$$

$$= P(X<3) \text{ as required.}$$

b) we know $P(X=3)$ is 0.1550

so over 60 trials the distribution of scores of 3 sixes can be described by

$Y = B(60, 0.155)$

and $P(Y \geqslant 12) = 1 - P(Y \leqslant 11)$

$= 1 - 0.7885$ (BIN.DIST(11, 60, 0.155, cumm))

$= \underline{\underline{0.2115}}$

c) This is simple:

600 throws with $p = \frac{1}{6}$ for each

so the estimate is $\frac{600}{6} = 100$ sixes.

d) Z = no of sixes rolled in 60 days (600 trials)

can be modelled as $Z = B(600, \frac{1}{6})$

But we will approximate this by a Normal distribution $N(np, (\sqrt{npq})^2)$

where np is the expected mean, $= 100$

p is the individual prob^y, $\frac{1}{6}$

$n = 600$

$q = (1-p)$

This means the Normal dist^n is $N\left(100, \sqrt{\left(\frac{250}{3}\right)}^2\right)$

Now we want to know $P(Z>95)$.

$B(600, \frac{1}{6})$ is a <u>discrete</u> distribution,

but $P(100, \frac{250}{3})$ is continuous. Because of

this we consider a value of 95.5 and

say

$P(Z>95)$ is got from N as

$P(Z>95.5)$ which is $1 - P(Z \leq 95.5)$

$= 1 - 0.3110$ (NORM.DIST $(95.5, 100, \frac{250}{3}$, cumul))

$= \underline{\underline{0.689}}$

2) a) The gradient of this line is -1.28 m/s, which indicates the bird leaves the nest on a continuous course downwards of 1.28 m/s (downwards - i.e dropping height)

b) Hypothesis H_1 : the real correlation $\rho < 0$
i.e there is negative correlation;
a one-tailed test.

r, the sample correlation, is measured at -0.510

And the sample size is 10

Look for a one-tailed sample at 5% significance

Using the 'Critical values for corr. coeffs' table, this gives a critical value of 0.5494.

So the measured value 0.510 is less than the critical value - so there is <u>not</u> sufficient evidence to support H_1.

c) a single line model doesn't do very well at all: it doesn't at all describe the initial rise in h, and it doesn't appear to come near the later decrease with its gradient of (about) −4 rather than −1.28.

d) $h = 38.1 - (t-k)^2$ is a parabola with max. 38.1 and axis at $t = k$.

From the diagram this suggests k should be about 3.5 – or more widely somewhere in $3.0 \leq k \leq 4.5$.

3) a) It's not immediately obvious from the LDS – and we don't have access anyway in the exam – but looking at it he seems to have used Rainfall instead of Air Temperature.

b) $$\text{Mean } \mu = \frac{\sum x}{n} = \frac{2801.2}{184} = \underline{\underline{15.2239}}$$

$$\text{SD } \sigma = \sqrt{\left(\frac{\sum x^2}{184} - \mu^2\right)} = \sqrt{\left(\frac{44695.4}{184} - 15.2239^2\right)} = \sqrt{11.1427} = \underline{\underline{3.3381}}$$

c) The mean here is higher than the overall mean, which relates to May – Oct.

And the SD is lower.

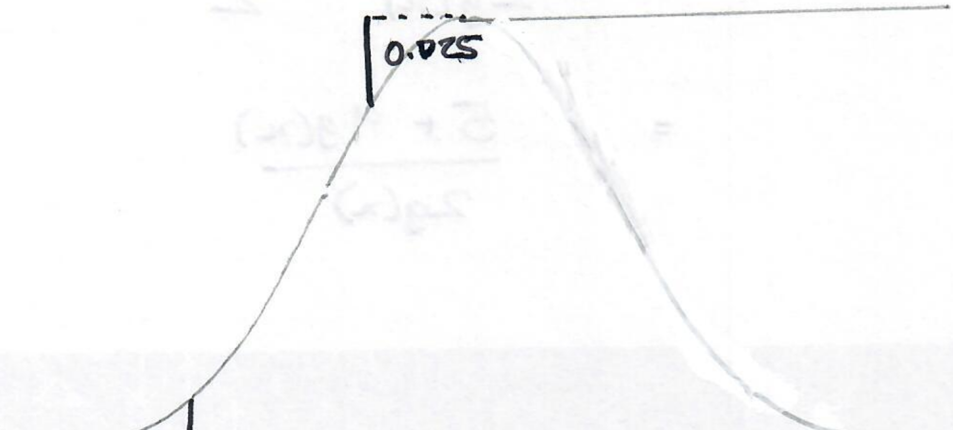
So we need to look for a month which is like 'summer'.

Perth summer is (something like) Nov-Feb, so the closest is Oct.

So we suggest the data is for October.

4) We're dealing with a binomial distribution $X \sim B(40, 0.1)$.

To find the critical region for a 5% level of significance with <u>two</u> tails, we look for the two ends with 0.025 cumulative probability:



Let H_0 be: p in the sample = 0.1

H_1 be: p ______ $\neq 0.1$

We look in the $B(40, \frac{1}{10})$ table and find

Likelihood of $0 = 0.0148$ ← 0.025

$0 - 1 = 0.0805$

⋮

$7 - 40 = 0.0995$

$8 - 40 = 0.0419$ ← 0.025

$9 - 40 = 0.0155$

So we set the critical values as 0 and 9, saying the significance of values ≤ 0 or ≥ 9 have a likelihood of $0.0148 + 0.0155 = 0.0303$ and so if Freya's test reaches these values then it is significant and H_1 is supported.

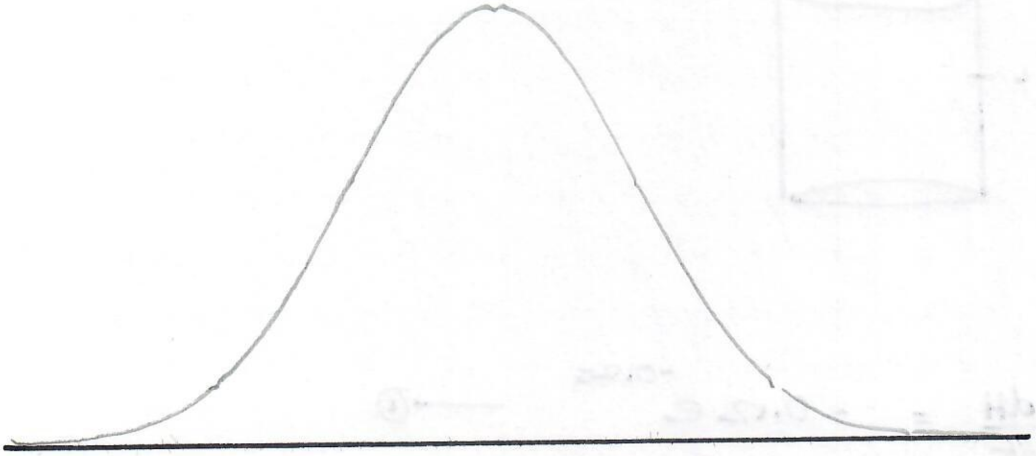
Put another way, the critical region is

$$\{X = 0\} \cup \{X \geq 9\}$$

b) As noted above, the actual significance level of this test is 0.0303

c) Freya's test result is 7, which is not in the critical region so H_1 is not supported.

5) a) $H = N(\underset{\mu}{1.4}, \underset{\sigma^2}{0.15^2})$



Let $X = \dfrac{H-\mu}{\sigma}$: then X is the standard normal distribution

$\dfrac{1.6-1.4}{0.15} = 1.333$

so we require $P(X > 1.333)$

$= 1 - P(X \leqslant 1.3333)$

$= 1 - 0.90879$ (by calc, since we don't have tables)

$= \underline{\underline{0.09121}}$

b) $T = N(330, 26^2)$

The head has to assume H and T are independent variables (which may not be true)

c) $P(T < \underset{min}{5}) = P(T < \underset{sec}{300}) = 0.12428$

So Probability of both $(H > 1.6)$ and $(T < 300)$

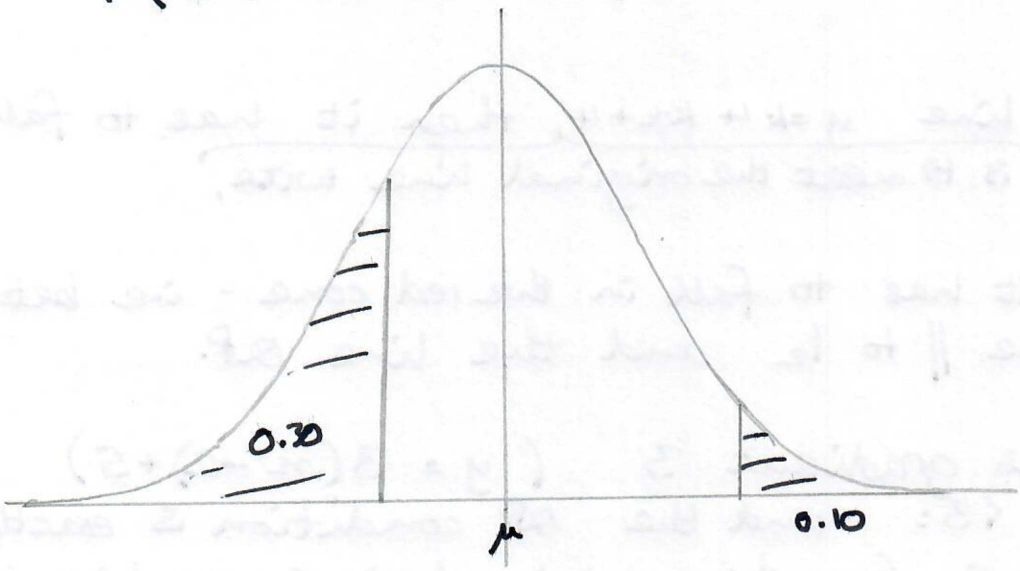
is $0.09121 \times 0.12428 = 0.01134$

i.e. 1.134% of the population, or 1 in 88.

d) $D \sim N(\mu, \sigma^2)$

$P(D < 16.3) = 0.30$

$P(D > 29.0) = 0.10$



Transfer D to the standard normal distⁿ $Z \sim N(0,1)$

by $Z = \dfrac{D - \mu}{\sigma}$

$D = 16.3 \Rightarrow Z = \dfrac{16.3 - \mu}{\sigma}$

$D = 29.0 \Rightarrow Z = \dfrac{29.0 - \mu}{\sigma}$

Now find the Z values which give P of 0.3 and 0.1:

-0.52440 (0.3)

1.28155 (1 − 0.1)

So $16.3 - \mu = -0.52440\,\sigma$ — ①

$29 - \mu = 1.28155\,\sigma$ — ②

② - ①:

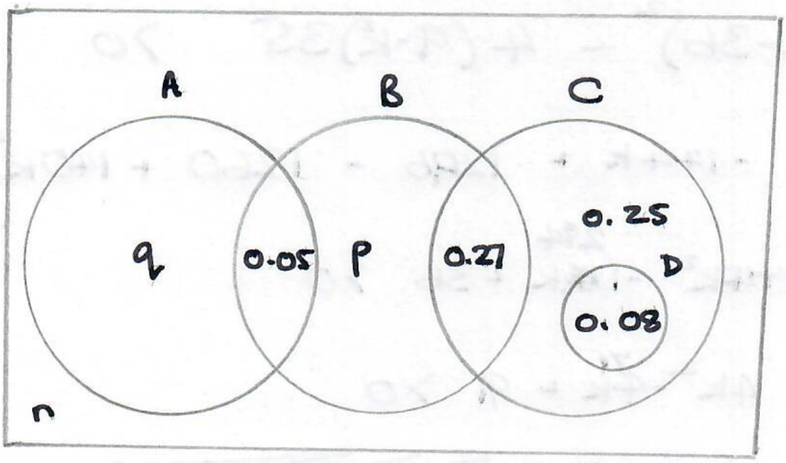
$12.7 = 1.80595\,\sigma$

$\sigma = 7.03231$

from ①: $\mu = 16.3 + 0.52440 \times 7.03231$

$= 19.98774.$

6)



a) mutually exclusive events could be

A, C

A, D

B, D

b) B and C are independent – meaning

$P(B \cap C) = P(B) . P(C).$

So $0.27 = (0.05 + p + 0.27)(0.27 + 0.25 + 0.08)$

$$0.27 = (0.32 + p)(0.6)$$

$$p + 0.32 = \frac{0.27}{0.6} = 0.45$$

$$\underline{\underline{p = 0.13.}}$$

c) (we note the total space must have a score/prob^y of 1).

$P(A|B') = q$ ~~$(q+r) - r$ i.e. q.~~

Since we know all the other values,

$$q + r = 1 - 0.05 - 0.13 - 0.27 - 0.25 - 0.08$$

$$= 1 - 0.78$$

$$= 0.22.$$

There's no lower bound on r other than 0, so q can be 0.22 at max.

d) $P(B|A') = 0.5$

$$P(A') = r + p + 0.27 + 0.25 + 0.08$$

$$= r + 0.13 + 0.27 + 0.25 + 0.08$$

$$= 0.73 + r$$

$$P(B) = \;\; + p + 0.27 = \;\; + 0.13 + 0.27$$

$$= 0.4$$

So $P(B|A') = \dfrac{P(B)}{P(A)} = \dfrac{0.4}{0.73+r} = 0.5$ (given)

$$\frac{0.8}{0.73+r} = 1$$

$$0.8 = 0.73+r$$

$$\underline{\underline{r = 0.07}}$$

and since we know $q + r = 0.22$,

$$q = 0.22 - 0.07 = \underline{\underline{0.15}}$$

e) $(A\cup B)' \cap C$ is the parts labelled 0.25 and 0.08 – so

$$P((A\cup B)' \cap C) = 0.25 + 0.08 = \underline{\underline{0.33}}$$

f) It's the intersection of B with the complement of $(A \cup C)$: so

$$\underline{B \cap [A\cup C]'}$$