1) $g(x) = 3x^3 - 20x^2 + (k+17)x + k$

$(x-3)$ is a factor, so we know 3 is a root and $g(3) = 0$

so $0 = 3 \times 27 - 20 \times 9 + 3(k+17) + k$

$= 81 - 180 + 51 + 3k + k$

$= -48 + 4k$

so $4k = 48$

$\underline{\underline{k = 12.}}$

(a remarkably easy 3 marks, unless you try to factorise it.)

2) a) $$(1-9x)^{\frac{1}{2}} = 1 + \frac{\frac{1}{2}}{1!}(-9x) + \frac{\frac{1}{2}(-\frac{1}{2})}{2!}(-9x)^2 + \frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{3!}(-9x)^3$$

(the general coeff is $\dfrac{\overbrace{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)\ldots}^{n \text{ terms}}}{n!}(-9x)^n$)

$$= 1 - \frac{9}{2}x - \frac{81}{8}x^2 - \frac{729}{16}x^3$$

b) The expansion of $(1-t)^{\frac{1}{2}}$ is only valid if $|1-t|<1$: so in this case $|9x|<1$ or $|x| < \frac{1}{9}$. Using $\frac{2}{9}$ violates this and will lead to a non-convergent sequence.

Curiously we can do better if we look for $\sqrt{\frac{1}{3}}$ and use $\left(1-9\times\frac{2}{27}\right)^{\frac{1}{2}} = \left(1-\frac{2}{3}\right)^{\frac{1}{2}} = \left(\frac{1}{3}\right)^{\frac{1}{2}}$.

Using $x = \frac{2}{27}$ gives (up to x^3) $\sqrt{\frac{1}{3}} \approx 0.5926$

so $(\sqrt{3})^2 \approx 2.8477$

(up to x^5) $\sqrt{\frac{1}{3}} \approx 0.5813$

so $(\sqrt{3})^2 \approx 2.9596$.

None of this is brilliant, though.

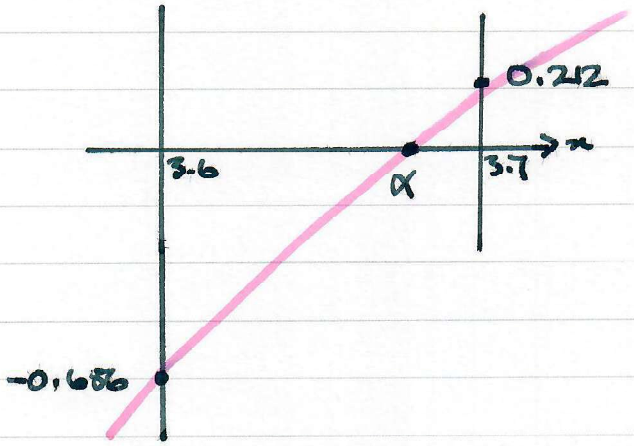
3) $f(x) = x + \tan\left(\frac{x}{2}\right) \qquad \pi < x < \frac{3\pi}{2}$

a) $f(3.6) = 3.6 + \tan(1.8) = -0.686$

$f(3.7) = 3.7 + \tan(1.85) = 0.212$

So given that there is only 1 root α, α must lie between $x = 3.6$ and 3.7

(assuming the function is continuous)



b) For the N-R method we need $f'(x)$:

$$f'(x) = 1 + \frac{1}{2}\sec^2\frac{x}{2} = 1 + \frac{1}{2\cos^2\frac{x}{2}}$$

and the NR formula is

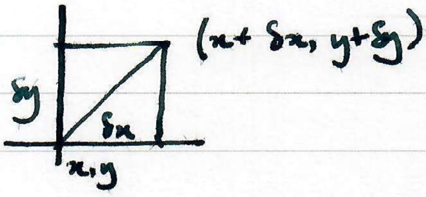
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

We draw a table:

n	x_n	$f(x_n)$	$f'(x_n)$	$x_n - \frac{f(x_n)}{f'(x_n)}$
		$= x + \tan\left(\frac{x}{2}\right)$	$= 1 + \frac{1}{2\cos^2\left(\frac{x}{2}\right)}$	
1	3.7	0.212	7.583	3.672
2	3.672	~~-0.0095~~ -0.01	8.278	3.673
3	3.673	+0.002	8.251	3.673

The N-R process has converged to give $\alpha = 3.673$.

4) $y = x^2$



Consider a point $(x + \delta x, y + \delta y)$:
The gradient of the chord is $\frac{\delta y}{\delta x}$.

Then $y + \delta y = (x + \delta x)^2$
$= x^2 + 2x\delta x + \delta x^2$

Subtract $y = x^2$:

$$\delta y = 2x\delta x + \delta x^2$$

As $\delta x \to 0$ the δx^2 term $\to 0$ more rapidly

and $\delta y \to 2x\delta x$

or $\frac{\delta y}{\delta x} \to 2x$

So in the limit, $\frac{dy}{dx} = 2x$. and this is the gradient of the chord.

(suspect my argument here is a bit scrappy)

5) $$f(x) = \frac{2x-3}{x^2+4}$$

Using the Division Rule $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$:

$$f'(x) = \frac{(x^2+4)2 - (2x-3)2x}{(x^2+4)^2}$$

$$= \frac{2x^2+8 - 4x^2 + 6x}{(x^2+4)^2}$$

$$= \frac{-2x^2+6x+8}{(x^2+4)^2}$$

$$= \frac{ax^2+bx+c}{(x^2+4)^2}$$

where $a = -2$, $b = 6$, $c = 8$

as required.

b) Since $(x^2+4)^2$ is always positive,

$f'(x)$ is $-$ve if and only if the numerator is $-$ve

So we require

$-2x^2+6x+8$ to be $-$ve for $f(x)$ to be decreasing.

this means

$$-2x^2+6x+8<0$$
$$x^2-3x-4>0$$
$$(x-4)(x+1)>0 \quad \text{———} \quad (*)$$

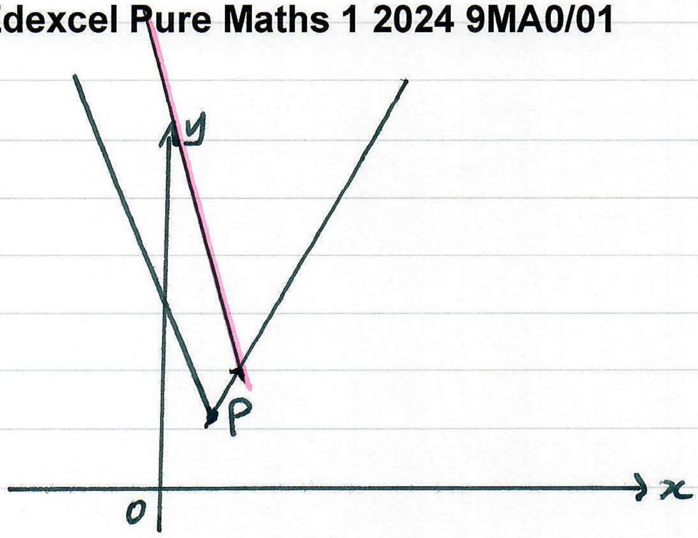
This has roots $x=-1$ and 4;

and is +ve for large x, so we can say this is <u>+ve</u> outside the roots - i.e. when

$$x<-1 \quad \text{or} \quad x>4.$$

Note it's $(*)$ we need to be true, so the requirement is <u>+ve</u> not <u>$-$ve</u>.

6)



$y = 3|x-2|+5$

a) The $|x-2|$ term clearly has a minimum when $x=2$, which means y has a minimum.

At this point $y = 3|2-2|+5 = 3\times 0+5 = 5$

So the minimum, at P, is $(2,5)$.

b) $16-4x = 3|x-2|+5$

Method ①: Consider the 2 lines (cases)

$16-4x = 3(x-2)+5$

$16-4x = 3x-6+5 = 3x-1$

$17 = 7x \qquad x = \frac{17}{7}$

$16-4x = 3(2-x)+5$

$16-4x = 6-3x+5 = -3x+11$

$5 = x \qquad x = 5$ (but this doesn't work)

Method 2: square the | | term:

$$16 - 4x = 3|x-2| + 5$$

$$(11 - 4x)^2 = 9|x-2|^2$$

$$121 - 88x + 16x^2 = 9x^2 - 36x + 36$$

$$7x^2 - 52x + 85 = 0$$

$$x = \frac{52 \pm \sqrt{52^2 - 7\times4\times85}}{14}$$

$$= \frac{52 \pm \sqrt{324}}{14}$$

$$= \frac{52 \pm 18}{14}$$

$$= \frac{70}{14} \text{ or } \frac{34}{14}$$

$$= 5 \text{ or } \frac{17}{7} \quad \text{as before.}$$

but this doesn't work.

It's worth noting $16-4x$ has gradient -4, so there can only be one solution: $x = \frac{17}{7}$. Whatever the other one represents, it's wrong.

c) $kx + 4 = 3|x-2| + 5$

Again $kx - 1 = 3|x-2|$

$$(kx-1)^2 = 9(x-2)^2$$

$$k^2x^2 - 2kx + 1 = 9x^2 - 36x + 36$$

$$0 = (9-k^2)x^2 + (-36+2k)x + 35$$

$$0 = (9-k^2)x^2 + (2k-36)x + 35$$

For this to have 2 distinct solutions the discriminant "$b^2 - 4ac$" must be +ve:

$$(2k-36)^2 - 4(9-k^2)35 > 0$$

$$4k^2 - 144k + 1296 - 1260 + 140k^2 > 0$$

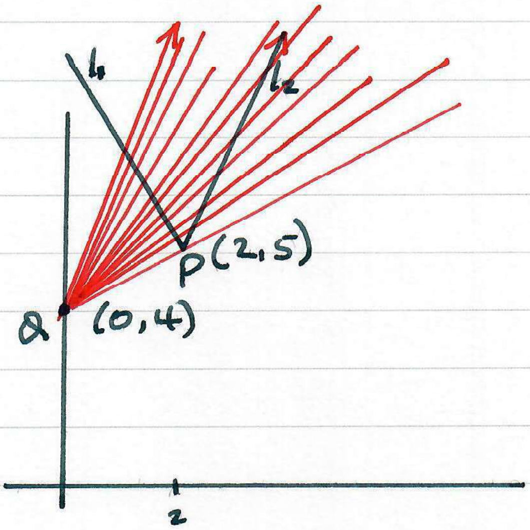
$$144k^2 - 144k + 36 > 0$$

$$4k^2 - 4k + 1 > 0$$

$$(2k-1)^2 > 0 \quad \text{so} \quad k > \tfrac{1}{2}.$$

But there's also another condition ...

The published answers don't make it clear, but if you draw this out:



If the line $y = kx+4$, then it has to fall
is to meet the original line twice,

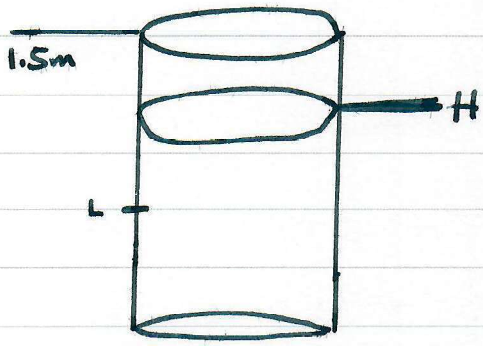
then it has to fall in the red cone - i.e. between the line $\parallel$ to l_2 and the line QP.

l_2 has gradient 3 $\quad (y = 3(x-2)+5)$
so $k < 3$: and the QP condition is exactly the one found: $k > \frac{1}{2}$ (since gradient $QP = \frac{1}{2}$).

So $\quad \frac{1}{2} < k < 3$ is the required condition.

<u>Moral</u>: Be very careful with $| \; |$ and squaring things.

7)



$$\frac{dH}{dt} = -0.12e^{-0.2t} \longrightarrow ①$$

a) ~~dH~~ Integrating ①:

$$H = -0.12\int e^{-0.2t}\,dt$$

$$= -0.12 \times \frac{1}{-0.2} e^{-0.2t} + C$$

$$= 0.6e^{-0.2t} + C$$

when $t=0$, $H = 1.5$, so

$$1.5 = 0.6 \times e^{0} + C = 0.6 + C$$

$$C = 0.9$$

So $H = 0.6e^{-0.2t} + 0.9$ $\left(= Ae^{-0.2t} + B \text{ where } A = 0.6,\ B = 0.9 \right)$

b) when $H = 1.2$m,

$$1.2 = 0.6e^{-0.2t} + 0.9$$

$$0.6e^{-0.2t} = 0.3$$

$$e^{-0.2t} = \frac{1}{2}$$

$$-0.2t = \ln\left(\frac{1}{2}\right)$$

$$t = \frac{1}{-0.2}\ln\left(\frac{1}{2}\right) = 3.466 \text{ hours}$$

$$= \underline{\underline{3h\ 28m}}$$

c) $H = 0.6e^{-0.2t} + 0.9$

In the long term $e^{-0.2t} = \frac{1}{e^{0.2t}}$ which $\rightarrow 0$,

so the 0.6 term $\rightarrow 0$ and 0.9 predominates.

So the model predicts $H \rightarrow 0.9$

i.e. the height of the hole is 0.9m.

8) $f(x) = 4 - 3x^2$

$g(x) = \dfrac{5}{2x-9}$

a) $fg(2) = f(g(2))$

$= f\left(\dfrac{5}{4-9}\right) = f\left(\dfrac{5}{-5}\right) = f(-1)$

$= 4 - 3(-1)^2 = 4 - 3 = \underline{\underline{1}}$

b) $g(x) = \dfrac{5}{2x-9}$

$\Rightarrow \quad 2x - 9 = \dfrac{5}{g(x)}$

$2x = \dfrac{5}{g(x)} + 9$

$x = \dfrac{5}{2g(x)} + \dfrac{9}{2}$

$= \dfrac{5 + 9g(x)}{2g(x)}$

Substituting ~~Exchanging~~ $g^{-1}(x)$ for x and x for $g(x)$

this gives

$$g^{-1}(x) = \frac{5+9x}{2x} \quad \text{or} \quad \frac{5}{2x} + \frac{9}{2}$$

where $x \neq 0$

$x \in \mathbb{R}$

c) i) $gf(x) = g(f(x)) = \dfrac{5}{2(4-3x^2)-9}$

$$= \frac{5}{8-6x^2-9}$$

$$= \frac{5}{-6x^2-1}$$

$$= \frac{-5}{1+6x^2}$$

ii) the <u>range</u> is the set of possible values,

which since $1+6x^2 \geqslant 1$ always

are in $[-5, 0)$ i.e. $-5 \leqslant gf(x) < 0$.

Here,

d) $h(x) = 2x^2 - 6x + k = 4 - 3x^2 = f(x)$

So $5x^2 - 6x + (k-4) = 0$

This has no real solutions when the discriminant ('$b^2 - 4ac$') is <0, i.e. when

$$6^2 - 4 \times 5 \times (k-4) < 0$$

$$36 - 20k + 80 < 0$$

$$116 < 20k$$

$$k > \frac{116}{20} = 5.8$$

So there are no real solutions when $\underline{k > 5.8}$.

a) Because the terms form a geometric sequence (u_1 u_2 u_3 etc) we can say

$$\frac{u_2}{u_1} = \frac{u_3}{u_2} = r = \frac{9^{7-2k}}{3^{4k-5}} = \frac{3^{2(k-1)}}{9^{7-2k}}$$

Replace 9 by 3^2:

$$\frac{3^{14-4k}}{3^{4k-5}} = \frac{3^{2k-2}}{3^{14-4k}}$$

Multiplying out:

$$3^{2(14-4k)} = 3^{(2k-2+4k-5)}$$

So equating indices:

$$28-8k = \cancel{8k}\ 6k-7$$

$$35 = 14k$$

$$\underline{\underline{k = \frac{5}{2}}}$$

This means

$u_1 = 3^{4k-5} = \underline{3^5}$

$u_2 = 9^{7-2k} = 9^2 = \underline{3^4}$

$u_3 = \underline{3^3}$

which is a geom. seq with $r = \frac{1}{3}$.

Using the usual (given) formula for the sum to ∞:

$$S_\infty = \frac{u_1}{1-r} = \frac{3^{4k-5}}{1-\frac{1}{3}}$$

$$= \frac{3^{10-5}}{2/3} = 243 \times \frac{3}{2}$$

$$= \underline{\underline{\frac{729}{2}}} \quad \text{or} \quad \underline{\underline{364.5}}$$

10) a) we could just plug $x=4$ in, but for a proper check on the roots:

$$0 = 8x - x^{\frac{5}{2}}$$

$$x^{5/2} = 8x$$

$$x\left(x^{3/2} - 8\right) = 0$$

So $x = 0$ giving point O

and $x^{\frac{3}{2}} = 8$: $x^3 = 64$: $x = 4$

(or other impossible values since $x \geq 0$) and is pres. in R.

So the x-coord of A is 4.

b) We need the gradient of the curve at A:

$$\frac{dy}{dx} = 8 - \frac{5}{2}x^{\frac{3}{2}} = 8 - \frac{5 \times 4^{3/2}}{2} = 8-20 = -12.$$

So the eq^n of the tangent l_1 is

$$y = -12x + c \quad \text{for some } c.$$

Subs. for the point $(4,0)$:

$$0 = -12 \times 4 + c = -48 + c: \qquad c = 48$$

So l_1 is $y = -12x + 48$

or $12x + y = 48$ as required. (1)

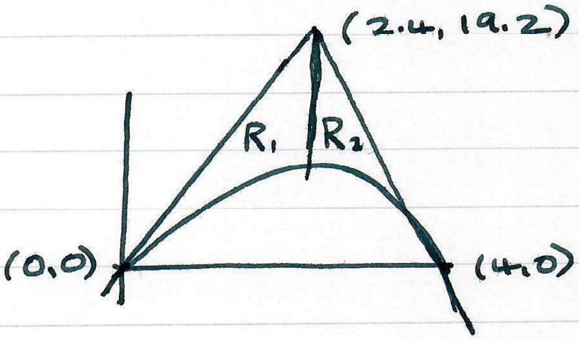
(2)

c) The line l_2 is $y = 8x$: so the two lines meet (subs (2) into (1)) at:

$$12x + 8x = 48$$
$$20x = 48$$
$$x = 2.4.$$

and at the intersection $y = 8x = 19.2$

So R is made up of two parts:



$$R_1 = \int_0^{2.4} 8x - (8x - x^{\frac{5}{2}})\,dx = \int_0^{2.4} x^{\frac{5}{2}}\,dx = \left[\frac{2}{7}x^{\frac{7}{2}}\right]_0^{2.4}$$

$$= \frac{2}{7}(2.4)^{\frac{7}{2}}$$

$$R_2 = \int_{2.4}^{4} (48-12x) - (8x - x^{\frac{5}{2}})\,dx$$

$$= \int_{2.4}^{4} 48 - 20x + x^{\frac{5}{2}}\,dx$$

$$= \left[48x - 10x^2 + \frac{2}{7}x^{\frac{7}{2}} \right]_{2.4}^{4}$$

$$= 48(4-2.4) - 10(16 - 2.4^2) + \frac{2}{7}\left(4^{\frac{7}{2}} - 2.4^{\frac{7}{2}}\right)$$

$$= 48 \times 1.6 \quad -10(16 - 5.76) + \frac{2}{7} \times 128 - \frac{2}{7}(2.4)^{\frac{7}{2}}$$

$$= 76.8 \quad - 10 \times 10.24 + \frac{256}{7} - \frac{2}{7}(2.4)^{\frac{7}{2}}$$

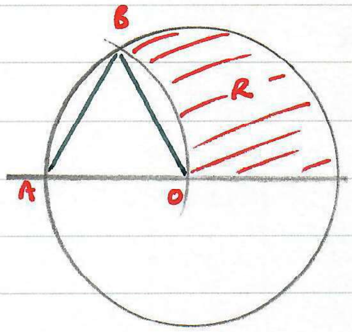
$$= 76.8 - 102.4 + \frac{256}{7} - \frac{2}{7}(2.4)^{\frac{7}{2}}$$

$$= 25.6 + \frac{256}{7} - \frac{2}{7}(2.4)^{\frac{7}{2}}$$

So $R = R_1 + R_2 = \cancel{\frac{2}{7}(2.4)^{\frac{7}{2}}} + 25.6 + \frac{256}{7} - \cancel{\frac{2}{7}(2.4)^{\frac{7}{2}}}$

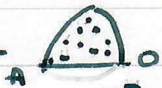
$$= 25.6 + \frac{256}{7} = \frac{256}{10} + \frac{256}{7} = \frac{1792 + 2560}{70} = \frac{4352}{70} = \frac{2176}{35}.$$

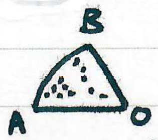
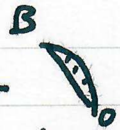
11.)



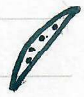
Diam = 10 cm

Radius = 5 cm.

We observe $R = (\text{half-circle}) -$ 

$= (\text{half-circle}) -$  $-$ 

(sector) (segment)

and segment OB  is equal to segment AB 

So: $\text{Half-circle} = \dfrac{\pi r^2}{2} = \dfrac{25\pi}{2}$

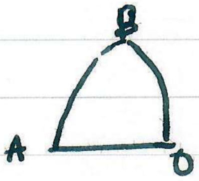
$\text{Sector } AOB = \dfrac{1}{6} \times \text{circle}$ (because AOB is an equilateral Δ so $\angle BOA = 60°$)

$= \dfrac{\pi r^2}{6} = \dfrac{25\pi}{6}$

$\Delta\ AOB = \dfrac{1}{2} \times \underset{(\text{base})}{5} \times \underset{(\text{height})}{\dfrac{5}{2}\sqrt{3}} = \dfrac{25\sqrt{3}}{4}$

Segment OB = segment AB = sector AOB − Δ AOB

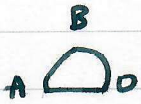
$= \dfrac{25\pi}{6} - \dfrac{25\sqrt{3}}{4}$

So 

$= \text{Sector}ABO + \text{Segment } OB$

$$= \frac{25\pi}{6} + \left(\frac{25\pi}{6} - \frac{25\sqrt{3}}{2}\right)$$

$$= \frac{25\pi}{3} - \frac{25\sqrt{3}}{4}$$

So R = half-circle − 

$$= \frac{25\pi}{2} - \frac{25\pi}{3} + \frac{25\sqrt{3}}{4}$$

$$= \frac{25\pi}{6} + \frac{25\sqrt{3}}{4}$$

$$= \frac{25}{4}\sqrt{3} + \frac{25\pi}{6} \qquad (\text{cm}^2)$$

which is in the required form $a\sqrt{3} + b\pi$

with $a = \frac{25}{4}$ and $b = \frac{25}{6}$

12) a) $I = 140\cos\theta - 480\sin\theta = K\cos(\theta + \alpha)$

Use the trigonometric identity

$$R(\cos A\cos B - \sin A \sin B) = R\cos(A+B)$$

and let $R\cos B = 140$

$R\sin B = 480$

Then $\dfrac{R\sin B}{R\cos B} = \tan B = \dfrac{480}{140} = \dfrac{24}{7} = 3.428$

so $\cancel{\tan}\ B = 1.287$ (radians)

$= 73.74^\circ$ (2dp)

And $R\sin(73.74^\circ) = 480$

$R = 480/\sin(73.74^\circ) = 500.01$ (2dp)

So $I = 140\cos\theta - 480\sin\theta$

$= 500.01\cos(\theta + 73.74^\circ)$

where $K = 500.01$ and $\alpha = 73.74^\circ$.

b) using the identity, we can say

$R = A + 500\cos(30t + 73.74)^\circ$

(dropping the 0.01)

b) i) Since t is the number of months the cos term will range through a whole 360° cycle in a year – so it'll reach its max (=1) and min (= −1).

So we can say

$$\max R = A + 500 \times 1 = 1500 \text{ (given)}$$

$$\text{So} \quad A = 1000$$

This means the equation for the model is:

$$R = 1000 + 500\cos(30t + 73.74)^\circ \quad \circledast$$

ii) This means the min R will occur when the cos term is −1, ie

$$\min R = 1000 + (500 \times -1) = 500.$$

c) We check this by putting $t = 3.5$ into $\circledast$:

$$R = 1000 + 500\cos(30 \times 3.5 + 73.74)^\circ$$

$$= 100 + 500\cos(105 + 73.74)^\circ$$

$$= 100 + 500\cos 178.74^\circ$$

$= 500.1$ which may be close enough for 'middle' of April

d) $F = 100 + 70\sin(30t + 70)^\circ$

This reaches a minimum when $\sin(30t + 70)^\circ = -1$,

i.e. $30t + 70 = 270$

$$t = \frac{200}{30} = \frac{20}{3} = 6\tfrac{2}{3}$$

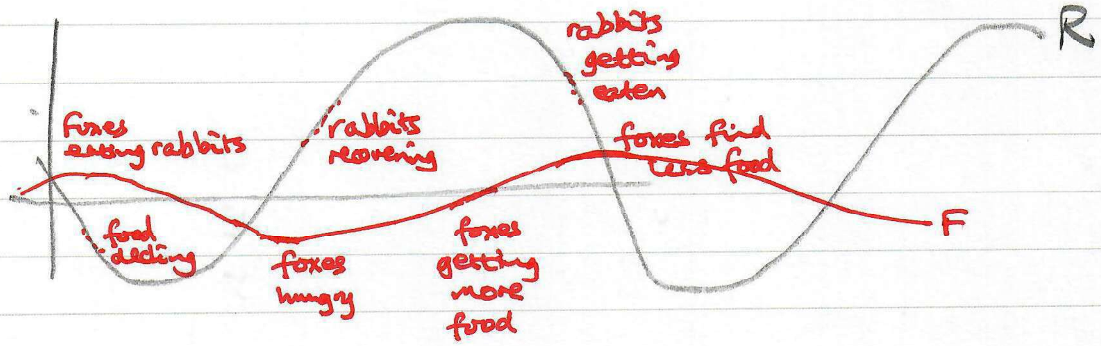
At this time

$$R = 1000 + 500\cos\left(\frac{30 \times 20}{3} + 73.74\right)^\circ$$

$$= 1000 + 500\cos 273.74^\circ$$

$$= 1033.$$

You might wonder how the curves interrelate — it's actually classic predator/prey, but the sin term in F is a bit confusing. It'd be better to express it as $\cos(-90 + 30t + 70)$, which would show the F population lagging the R:



13) $$I = \int_0^a x^{\frac{1}{2}}\sqrt{a-x}\; dx$$

Let $x = a\sin^2\theta$: limits are $x=0 = a\sin^2\theta : \theta = 0$

$a - x = a(1-\sin^2\theta) = a\cos^2\theta$ $\qquad x = a = a\sin^2\theta : \theta = \frac{\pi}{2}$

Then $\dfrac{dx}{d\theta} = a\,2\sin\theta\cos\theta\; d\theta$

And $$I = 2a\int_0^{\frac{\pi}{2}} \sqrt{a}\,\sin\theta\sqrt{a\cos^2\theta}\;\sin\theta\cos\theta\; d\theta$$

$$= 2a^2\int_0^{\frac{\pi}{2}} \sin^2\theta\cos^2\theta\; d\theta$$

$$= \frac{a^2}{2}\int_0^{\frac{\pi}{2}} 2\sin^2 2\theta\; d\theta$$

Now $\cos 2\phi = \cos^2\phi - \sin^2\phi = 1 - 2\sin^2\phi$

So $\sin^2\phi = \frac{1}{2}(1-\cos 2\phi)$

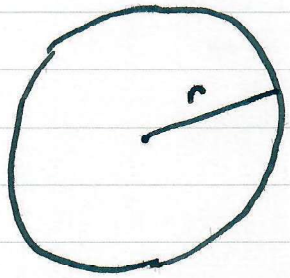
So $$I = \frac{1}{2\times 2}a^2\int_0^{\frac{\pi}{2}} 1 - \cos 4\theta\; d\theta$$

$$= \frac{a^2}{4}\left\{\theta - \frac{1}{4}\sin 4\theta\right\}_0^{\frac{\pi}{2}}$$

$$= \frac{a^2}{4}\left[\frac{\pi}{2} - \cancel{\frac{1}{4}\sin 2\pi} - 0 + \frac{1}{4}\sin 2\pi\right]$$

$= \frac{\pi}{8}a^2$ which is $k\pi a^2$ with $k = \frac{1}{8}$, as required.

14)



a) $\frac{dr}{dt} = k\frac{1}{\sqrt{r}}$ for some k.

(this is what we're told)

b) Solving the equation by separating variables:

$$\int \sqrt{r}\,dr = \int k\,dt$$

$$\frac{r^{\frac{3}{2}}}{3/2} = kt + c$$

$$r^{\frac{3}{2}} = \frac{3}{2}kt + \frac{3}{2}c$$

k and c are unknown constants so we can simplify:

$$r^{\frac{3}{2}} = Kt + C \quad ③$$ (this may not help in the long run)

Using the conditions when $t = 10$:

① $16^{\frac{3}{2}} = 10K + C$: $64 = 10K + C$

② $0.9 = k\frac{1}{4} = \frac{2}{3}\frac{K}{4} = \frac{K}{6}$

So $K = 6 \times 0.9 = 5.4$

From ①: $64 = 10 \times 5.4 + C = 54 + C$: $C = 10$

So in ③: $r^{3/2} = 5.4t + 10$ as required.

c) when $t = 20$,

$r^{3/2} = 5.4 \times 20 + 10 = 10.8 + 10 = 20.8$

So $r = \left(118\right)^{2/3} = 24.058$ cm

$= 241$ mm to the nearest mm.

d) 1. The model probably doesn't mirror the actual physics.

2. At some point the balloon will burst – or something.

15) (i) $k^2 - 4k + 5 = (k-2)^2 + 1$ (by completion of squares)

and this is always strictly +ve ($\geqslant 1$) for all real k.

So the expression is +ve ($\forall\ k \in R$) as required.

(ii) There's a neater proof of this by saying

let $z = \frac{x}{y}$ then $(3z+2)(2z-5) = 28$

$$6z^2 - 11z - 10 = 28$$

$$6z^2 - 11z - 38 = 0$$

$$z = \frac{11 \pm \sqrt{121 + 6 \times 4 \times 38}}{12}$$

$$= \frac{11 \pm \sqrt{1033}}{12}$$

The surd term is not a perfect square and so must be irrational ... so $\frac{x}{y}$ is irrational,

so at least one of x and y must be irrational... etc.

Still, here we go.

The student's proof considers

$$3x + 2y = A \quad \text{——— (1)}$$
$$2x - 5y = B \quad \text{——— (2)}$$

where $(A, B) = (14, 2)$. We will consider the other 5 cases where $(A, B) = (28, 1), (7, 4), (4, 7), (2, 14)$ and $(1, 28)$ – and also the other 6 cases where the factors could both be negative.

We can dismiss the latter 6 because if x and y are both +ve integers then the factor A cannot be -ve, and neither can B.

So we'll have 5 sets of 2 simultaneous equations to solve, with the same coeffs of x and y. We can use (1) and (2) to say:

$2 \times (1)$: $\quad 6x + 4y = 2A \quad (3)$

$3 \times (2)$: $\quad 6x - 15y = 3B \quad (4)$

$(4) - (3)$: $\quad -19y = 3B - 2A$

$$y = \frac{2A - 3B}{19}$$

In the 5 cases this gives:

$(28,1) \quad y = \frac{2 \cdot 28 - 3 \cdot 1}{19} = \frac{53}{19} \quad \text{not} \in \mathbb{N}$

$(7,4) \quad y = \frac{2 \cdot 7 - 3 \cdot 4}{19} = \frac{2}{19} \quad \text{not} \in \mathbb{N}$

$(4,7) \quad y = \frac{2 \cdot 4 - 3 \cdot 7}{19} = \frac{-13}{19} \quad \text{not} \in \mathbb{N}$

$(2,14) \quad y = \frac{2 \cdot 2 - 3 \cdot 14}{19} = \frac{-38}{19} = -2$

not +ve.

$(1,28) \quad y = \frac{2 \cdot 1 - 3 \cdot 28}{19} = -\frac{82}{19} \quad \text{not} \in \mathbb{N}$

None of these values for y is both integer and positive.

So there are no possible factors for the expression – hence the ~~proof~~ analysis contradicts the 'assumption', and the result is proven by contradiction.