1) a) use $v = u + at = 0 + 3.2 \times 5 = \underline{\underline{16 \text{ m/s}}}$.

b) use $s = ut + \frac{1}{2}at^2 = \frac{3.2}{2} \times 5^2 = 1.6 \times 25 = \underline{\underline{40\text{m}}}$

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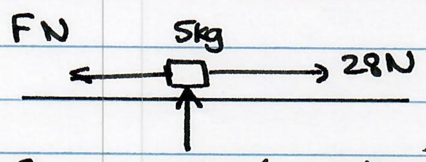
2)



a) $5 \times 9.8 = \underline{\underline{49\text{N}}}$ (reaction) (since the forces balance) vertically

b) use $F = ma$ horizontally:

$$(28 - F) = 5 \times 1.4 = 7$$

$$F = 28 - 7 = \underline{\underline{21 \text{ N}}}.$$

c) so $\mu = \frac{F}{\text{reaction}} = \frac{21}{49} = \underline{\underline{\frac{3}{7}}}$ (0.43)

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3) a) when $t=0$, $\underline{v} = 7\underline{i} - 3\underline{j}$ which has magnitude (= speed) $\sqrt{7^2 + 3^2} = \sqrt{50}$

$= 5\sqrt{2}$ or 7.6 m/s.

b) when P is moving $\parallel$ to $\underline{i} + \underline{j}$,

and given $\underline{v} = (t^2 - 3t + 7)\underline{i} + (2t^2 - 3)\underline{j}$

we must have (coefficients are in ratio):

$$\frac{t^2 - 3t + 7}{2t^2 - 3} = \frac{1}{1}$$

$$t^2 - 3t + 7 = 2t^2 - 3$$

$$0 = t^2 + 3t - 10$$

$$(t+5)(t-2) = 0$$

So $\underline{\underline{t = 2}}$ or -5 (discarded) seconds.

c) $\underline{a} = \frac{d\underline{v}}{dt} = (2t-3)\underline{i} + 4t\underline{j}$

($= \underline{i} + 8\underline{j}$ when $t = 2$, though apparently this isn't needed)

d) You might think of this as meaning

$$\frac{4t}{2t-3} = \infty, \quad \text{so} \quad 2t - 3 = 0.$$

(or say - if $\underline{a}$ is perpendicular to $\underline{i}$, then $\underline{a} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ must be 0)

so $t = \frac{3}{2} = \underline{\underline{1.5s.}}$

4) a) $\underline{a} = 2.4\underline{i} + \underline{j}$

So $\underline{v} = (2.4t + c_i)\underline{i} + (t + c_j)\underline{j}$

a) when $t = 0$, at point A:

$\underline{v} = c_i\underline{i} + c_j\underline{j}$

$= -16\underline{i} - 3\underline{j}$

so $c_i = -16$, $c_j = -3$

and in general, $\underline{v} = (2.4t - 16)\underline{i} + (t - 3)\underline{j}$

So at B: $\underline{v} = (2.4 \times 5 - 16)\underline{i} + (5 - 3)\underline{j}$

$= -4\underline{i} + 2\underline{j}$

So speed at B $= \sqrt{4^2 + 2^2} = \sqrt{20} = 2\sqrt{5}$ or 4.47 m/s.

b) $\underline{v} = (2.4t - 16)\underline{i} + (t - 3)\underline{j}$

so $\underline{s} = (1.2t^2 - 16t + d_i)\underline{i} + \left(\frac{t^2}{2} - 3t + d_2\right)\underline{j}$

when $t = 0$, at point A:

$\underline{s} = d_i\underline{i} + d_j\underline{j} = 44\underline{i} - 10\underline{j}$

So $d_i = 44$, $d_j = -10$

so $\underline{s} = (1.2t^2 - 16t + 44)\underline{i} + \left(\frac{t^2}{2} - 3t - 10\right)\underline{j}$

Substitute for T at point C: $(4\underline{i} + c\underline{j})$

$$1.2T^2 - 16T + 44 = 4 \quad \text{——} \quad ①$$

$$\frac{T^2}{2} - 3T - 10 = c \quad \text{——} \quad ②$$

① gives: $1.2T^2 - 16T + 40 = 0$

$$6T^2 - 80T + 200 = 0$$

$$(3T - 10)(2T - 20) = 0$$

So $T = \frac{10}{3}$ (invalid) or $\underline{\underline{T = 10}}$.

c) And in ②: $c = \frac{100}{2} - 30 - 10 = \underline{\underline{10}}$

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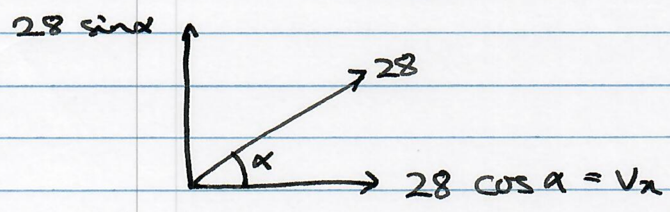
5) Resolve the velocity components:



Horizontally there's no acceleration so the speed is constant. So:

$$TV_x = 40$$

$$\underline{\underline{T = \frac{40}{28\cos\alpha}}} \quad \text{as required.}$$

b) Vertically, use "$s = ut + \frac{1}{2}at^2$":

$$20 = 28\sin\alpha \times T - \frac{1}{2}9.8T^2$$

$$= \frac{40 \cdot 28\sin\alpha}{28\cos\alpha} - \frac{1}{2}\,9.8\left(\frac{40}{28\cos\alpha}\right)^2$$

$$= 40\tan\alpha - 10\sec^2\alpha$$

$$= 40\tan\alpha - 10 - 10\tan^2\alpha$$

$$10\tan^2\alpha - 40\tan\alpha + 30 = 0$$

so $\underline{\underline{\tan^2\alpha - 4\tan\alpha + 3 = 0}}$ as required.

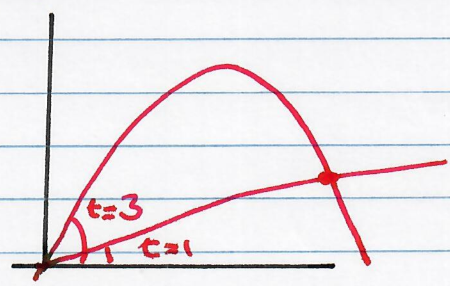
c) At this point it's useful to solve the equation:

$$t^2 - 4t + 3 = 0$$ ($t = \tan\alpha$, not time, here)

$$(t-3)(t-1) = 0$$

$\tan\alpha = t = 1$ or 3.

Both values would work, but we're asked for 'the greatest possible height, which occurs when $\tan\alpha = 3$.



So $\sin\alpha = \dfrac{3}{\sqrt{10}}$

We have $y = 28 \sin\alpha\, t - 4.9t^2$

$$= \frac{28 \times 3}{\sqrt{10}} t - 4.9t^2$$

So $\frac{dy}{dt} = \frac{28 \times 3}{\sqrt{10}} - 9.8t = 0$ for the highest point.

So $t = \frac{84}{9.8\sqrt{10}}$

and $y = 28 \times \frac{3}{\sqrt{10}} \times \frac{84}{9.8\sqrt{10}} - 4.9\left(\frac{84}{9.8\sqrt{10}}\right)^2$

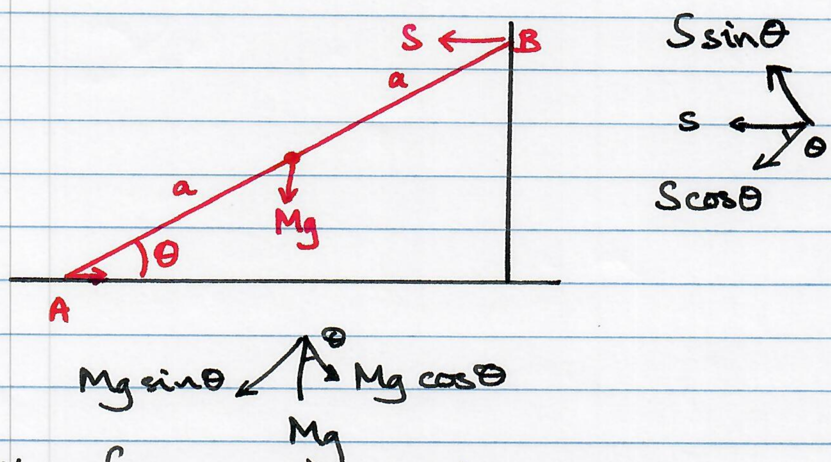
$= \cancel{4.9\left(\frac{84}{\ }\right.}$

$$= \frac{28 \times 3 \times 28 \times 3}{9.8 \times 10}\left(1 - \frac{1}{2}\right)$$

$$= \underline{\underline{36\text{m}.}}$$

d) The model doesn't allow for wind, spin ... and it assumes the ball is a point.

6)



Resolving the forces: see diagrams.

a) Since the rod is in equilibrium the horizontal forces on it are in balance: so since S acts leftwards, the frictional force at A must act to the right.

b) Taking moments about A, assuming the grav. force acts at the midpoint, and resolving it and S:

$$Mga\cos\theta = S.2a\sin\theta$$

So $S = \dfrac{Mga\cos\theta}{2a\sin\theta} = \underline{\underline{\tfrac{1}{2}Mg\cot\theta}}$ as required.

c) Matching the vertical forces, the normal (upwards) reaction at A equals Mg.

So the frictional force at A is μMg.

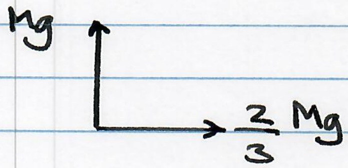
Matching the horizontal forces,

$$\mu Mg = S = \tfrac{1}{2}Mg\cot\theta$$

$$\cot\theta = \tfrac{4}{3} \text{ (given)}$$

So $\mu = \frac{1}{2}\cot\theta = \underline{\underline{\frac{2}{3}}}$

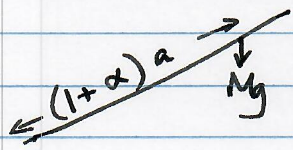
d) At A, we know the forces are:



So the magnitude of the resultant is:

$$\sqrt{1+\left(\frac{2}{3}\right)^2}\, Mg = \underline{\underline{\frac{1}{3}\sqrt{13}\, Mg}}.$$

e) With the centre of mass closer to B, the 'moment' equation is the same but we take the weight as acting at a point higher up the rod ... say at $(1+\alpha)a$



and the moment equation is

$$Mg\,(1+\alpha)a\cos\theta = S.\,2a\sin\theta$$

So $S = \frac{Mg}{2}(1+\alpha)\cot\theta$

So S is $\underline{\underline{\text{greater}}}$ than before.