Edexcel A Level Pure Maths 1 2023

1) $\int \frac{x^{\frac{1}{2}}(2x-5)}{3}\,dx$

$$= \int \frac{2}{3}x^{\frac{3}{2}} - \frac{5}{3}x^{\frac{1}{2}}\,dx$$

$$= \frac{2}{3}\cdot\frac{2}{5}x^{\frac{5}{2}} - \frac{5}{3}\cdot\frac{2}{3}x^{\frac{3}{2}} + C$$

$$= \frac{4}{15}x^{\frac{5}{2}} - \frac{10}{9}x^{\frac{3}{2}} + C$$

This looks to be 'each term in its simplest form', but it could be written as:

$$\frac{2}{3}x^{\frac{3}{2}}\left(\frac{2}{5}x - \frac{5}{3}\right) + C$$

2) (This question includes a useful condition, easily overlooked.)

$$f(x) = 4x^3 + 5x^2 - 10x + 4a$$

$(x-a)$ is a factor of $f(x)$, so $f(a)=0$:

$$4a^3 + 5a^2 - 10a + 4a = 0$$

$$4a^3 + 5a^2 - 6a = 0$$

so (i): $a(4a^2 + 5a - 6) = 0$

$$a(a+2)(4a-3) = 0$$

So $a = +\frac{3}{4}$, 0 or -2: but we are told a is positive

So we discount $a=0$ and $a=-2$ and take $a=\frac{3}{4}$.

If $f(x) = 3$ then

$$f(x) = 4x^3 + 5x^2 - 10x + \frac{4.3}{4} = 3$$

$$f(x) = 4x^3 + 5x^2 - 10x + \cancel{3} = \cancel{3}\,0$$

So $4x^3 - 5x^2 - 10x = 0$

$$x\,(4x^2 - 5x - 10) = 0$$

The quadratic term doesn't factorise neatly,

so $x = 0$ or $\dfrac{5 \pm \sqrt{25 + 4.4.10}}{2.4}$

i.e. 0 or $\frac{1}{8}\left(5 \pm \sqrt{185}\right)$

3) A is $5\underline{i} + 3\underline{j} + 2\underline{k}$

B is $2\underline{i} + 4\underline{j} + a\underline{k}$

a) $|\vec{OA}| = \sqrt{5^2 + 3^2 + 2^2} = \sqrt{25+9+4} = \sqrt{38}$

b) $|\vec{OB}| = \sqrt{2^2 + 4^2 + a^2} = \sqrt{4+16+a^2} = \sqrt{20+a^2}$

or

$$\alpha^2 - 8\alpha - 2 = 0$$

So $\alpha = \dfrac{8 \pm \sqrt{64+4.2}}{2} = \dfrac{8 \pm \sqrt{72}}{2}$

$= 4 \pm \sqrt{18}$

Discount $4+\sqrt{18}$ because we know α is small:

$\alpha = 4 - \sqrt{18} = \underline{\underline{-0.243 \text{ (rad)}}}$

to 3 dp.

b) $P(0,3)$ lies on C

The tangent $y = mx + c$ has gradient m,

and we know $m = f'(0) = 2x + \frac{1}{2}\cos x$

$= 2.0 + \frac{1}{2}.\cos 0$

$= \frac{1}{2}\cos 0$

$= \frac{1}{2}$

So the tangent line is $y = \frac{1}{2}x + c$

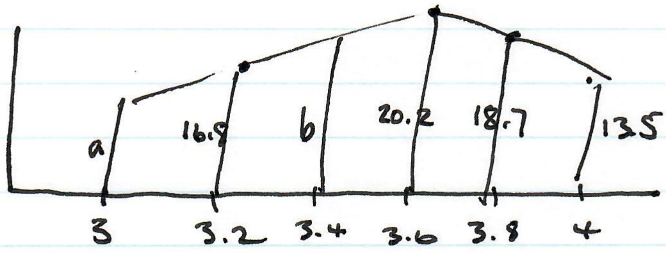
Substitute in for the point $P(0,3)$:

$3 = \frac{1}{2}.0 + c$

$3 = c$

So the tangent at P is $\underline{\underline{y = \frac{1}{2}x + 3}}$.

5)



a)

Trapezium rule approximates the area as:

$$0.2\left(\frac{a+16.8}{2} + \frac{16.8+b}{2} + \frac{b+20.2}{2} + \frac{20.2+18.7}{2} + \frac{18.7+13.5}{2}\right)$$

$$= \left(\frac{a}{2} + 16.8 + b + 20.2 + 18.7 + \frac{13.5}{2}\right)0.2$$

$$= \left(\frac{a}{2} + b + 62.45\right)0.2$$

$$\begin{array}{r} 16.8 \\ 20.2 \\ 18.7 \\ 6.75 \\ \hline 62.45 \end{array}$$

So given the area is 17.59 we can say:

$$\frac{a}{2} + b + 62.45 = 5 \times 17.59$$

$$= 87.95$$

So $\frac{a}{2} + b = \begin{array}{r} 87.95 \\ -\ 62.45 \\ \hline 25.50 \end{array} = 25.50$

So $a + 2b = 25.50 = 51.2$ as required.

b) we also know sum of all y values $= 97.2$

$= a + 16.8 + b + 20.2 + 18.7 + 13.5$

$= a + b + 69.2$

So $a + b = 97.2 - 69.2 = 38$

So we have:

$$a + 2b = 51 \qquad (1)$$

$$a + b = 35 \qquad (2)$$

(1) - (2): $b = 51 - 35 = 16$

subs. in (2): $a + 16 = 35$

$$a = 35 - 16 = 19.$$

So $\underline{a = 19, \quad b = 16}$ as required.

6) $a = \log_2 x \qquad b = \log_2 (x+8)$

a) $\log_2 \sqrt{x} = \log_2 x^{\frac{1}{2}} = \frac{1}{2} \log_2 x = \underline{\underline{\frac{a}{2}}}$

b) $\log_2 (x^2 + 8x) = \log_2 \{x.(x+8)\} = \log_2 x + \log_2 (x+8)$

$= \underline{\underline{a + b.}}$

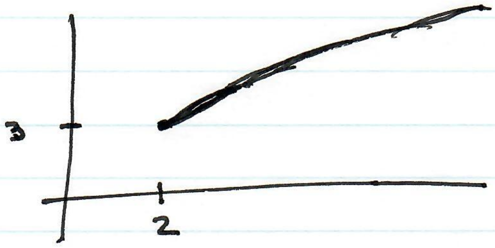
c) $\log_2 \left(8 + \frac{64}{x}\right) = \log_2 \left(\frac{8x + 64}{x}\right)$

$= \log_2 \frac{8(x+8)}{x} = \log_2 8 + \log_2 (x+8) - \log_2 x$

$= \underline{\underline{3 + b - a.}}$

7) $f(x) = 3 + \sqrt{x-2}$ $\qquad x \in \mathbb{R},\ x > 2.$

a) The range of f is the set of values f can take



and is $(3, \infty)$

(because x is strictly > 2)

b) $f(x) = 3 + \sqrt{x-2}$

So $\sqrt{x-2} = f(x) - 3$

$x - 2 = \sqrt{f(x) - 3}$

$x = \sqrt{f(x) - 3} + 2$

so $f^{-1}(x) = \sqrt{x - 3} + 2.$

c) $f(6) = 3 + \sqrt{6-2} = 3 + \sqrt{4} = 3 + 2 = 5$

$g(5) = \dfrac{15}{5-3} = \dfrac{15}{2}$

So $gf(6) = \dfrac{15}{2}$

d) if $f(a^2+2) = g(a)$

then $3 + \sqrt{a^2-2+2} = \dfrac{15}{a-3}$

so $3 + \sqrt{a^2} = \dfrac{15}{a-3}$

taking sq. root (and noting a could be ±ve)

$$3 + a = \frac{15}{a-3}$$

$$(3+a)(a-3) = 15$$

$$a^2 - 9 = 15$$

$$a^2 = 24$$

$$a = \pm 2\sqrt{6}$$

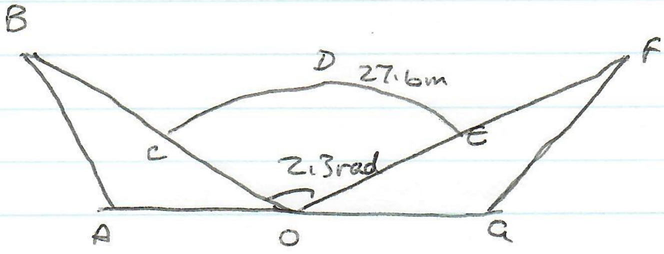
We check both +ve and -ve roots and find

$f(a^2+2) \neq g(a)$ when $a = -2\sqrt{6}$,

so the (one) exact answer is

$$\underline{\underline{a = 2\sqrt{6}}}$$

8)



(a) OC is the radius of the circle segment CDEO, so we know arc CDE = $OC \times 2.3$

i.e. $27.6\,m = OC \times 2.3$

$OC = \frac{27.6\,m}{2.3} = 12\,m$

(b) AOG is a straight line, so $\angle AOG = \pi$ rad i.e 3.142 rad.

$\triangle s$ AOB and GOF are congruent, so $\angle AOB = \angle GOF$.

So $A\hat{O}B + C\hat{O}E + E\hat{O}G = 3.142$

$2\,A\hat{O}B + 2.3 = 3.142$

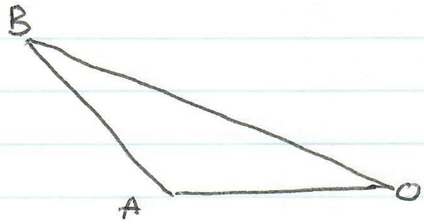
$A\hat{O}B = \frac{3.142 - 2.3}{2} = 0.421$ rad to 3dp.

(c) Front of the stage BCDEF = 35m.

So $BC = EF = (BCDEF - 27.6\,m)/2$
$= (35\,m - 27.6\,m)/2$
$= 7.4\,m/2$
$= 3.7\,m.$

So $BO = FO = \underset{BC}{3.7\,m} + \underset{CO}{12\,m} = 15.7\,m.$

Consider $\triangle$ BAO:



we know $BO = 15.7m$

$AO = 15m/2 = 7.5m$

and $A\hat{O}B = 0.421$ rad. $= 24.122°$

Using the standard formula for area of a $\triangle$,

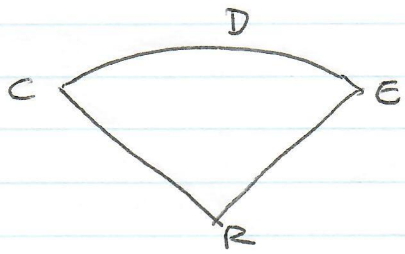
$$\triangle ABO = \frac{1}{2} BO . AO . \sin A\hat{O}B$$

$$= \frac{1}{2} . 15.7 . 7.5 \times \sin 24.122°$$

$$= \frac{1}{2} \times 15.7 \times 7.5 \times 0.4087$$

$$= 24.061 \; m^2$$

Now consider CDEO:



$$\text{Area of a segment} = \frac{\text{radius}^2 \times C\hat{R}E}{2}$$

$$= \frac{12^2 \times 2.3}{2} = 165.6 \; m^2.$$

So total area of stage $= 2 \times \triangle AOB + CDER$

$= 2 \times 24.061 + 165.6 \; m^2$

$= 48.122 + 165.6 m^2$

$= 213.722 m^2$

a) $u_1 = 3k+4 \qquad u_2 = 12-3k \qquad u_3 = k+16$

$\frac{u_2}{u_1} = \frac{u_3}{u_2} = r$, so

$\frac{12-3k}{3k+4} = \frac{k+16}{12-3k}$

$(12-3k)^2 = (3k+4)(k+16)$

$144 - 72k + 9k^2 = 3k^2 + 52k + 64$

$6k^2 - 124k + 80 = 0$

$3k^2 - 62k + 40 = 0$ as required.

$(3k-2)(k-20) = 0$

So $k = \frac{2}{3}$ or $k = 20$.

$k = \frac{2}{3}$ gives $r = \frac{12-3k}{3k+4} = \frac{12-2}{2+4} = \frac{10}{6} = \frac{5}{3}$

$k = 20$ gives $r = \frac{12-3k}{3k+4} = \frac{12-60}{60+4} = -\frac{48}{64} = -\frac{3}{4}$.

If the series is to converge, $|r| < 1$

so $k = 20$ is the only solution.

$S_\infty = a_1 \frac{1}{1-r}$

$a_1 = 3k+4 = 64$, so

$S_\infty = \frac{64}{1-(-\frac{3}{4})} = \frac{4 \times 64}{7} = \frac{256}{7} = 36\frac{4}{7}$.

10) a) C: $x^2+y^2+6kx-2ky+7=0$ ①

By a standard result, centre (p,q) is $(-3k,k)$

and radius is $\sqrt{p^2+q^2-r}$

$$=\sqrt{3k^2+k^2-7} = \sqrt{10k^2-7}$$

Check: $(x+3k)^2+(y-k)^2 = 10k^2-7$

Expand: $x^2+6kx+9k^2+y^2-2ky+k^2 = 10k^2-7$

$$\Rightarrow x^2+y^2+6kx-2ky+\cancel{10k^2} = \cancel{10k^2}-7$$

$$x^2+y^2+6kx-2ky+7=0$$

which matches ①

b) At the intersection points $y=2x-1$

So in ①:

$$x^2+(2x-1)^2+6kx-2k(2x-1)+7=0$$

$$x^2+4x^2-4x+1+6kx-4kx+2k+7=0$$

$$5x^2+(2k-4)x+2k+8=0$$

For this to have solutions the discriminant >0:

$$(2k-4)^2-4.5.(2k+8)>0$$

$$4k^2-16k+16-40k-160>0$$

$$4k^2-56k-144>0$$

$$k^2 - 14k - 36 > 0$$

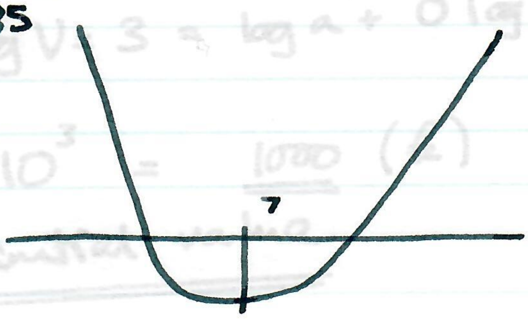
The roots of this expression are:

$$k = \frac{14 \pm \sqrt{14^2 + 4.36}}{2}$$

$$14^2 = 196$$
$$\frac{144}{340}$$

$$= 7 \pm \tfrac{1}{2}\sqrt{340}$$

$$= 7 \pm \sqrt{85}$$



So the conditions are

$\underline{\underline{k < 7 - \sqrt{85}}}$ or $\underline{\underline{k > 7 + \sqrt{85}.}}$

12) $y = \sin x$

From 1st principles, we say

$$\frac{dy}{dx} = \frac{\delta y}{\delta x} \quad \text{where} \quad \delta y = \frac{y(x+\delta x) - y(x)}{\delta x}$$

$$\text{So } \frac{\delta y}{\delta x} = \frac{\sin(x+\delta x) - \sin x}{\delta x}$$

Using standard formulae (and switching δx to h)

$$\text{this} = \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \sin x \left(\frac{\cos h - 1}{h} \right) + \cos x \frac{\sin h}{h}$$

using the approximations as $h \to 0$

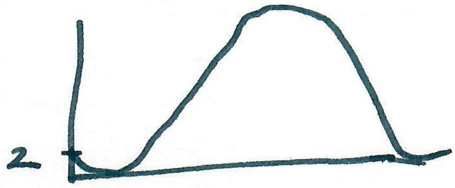
this meand $\frac{\delta y}{h}$ as $h \to 0$

$$\text{So } \lim_{h \to 0} \longrightarrow \sin x(0) + \cos x(1)$$

$$= \cos x.$$

$$\text{So } \frac{dy}{dx} = \cos x.$$

13) This is another 'stupid' roller coaster question where the 'model', if correct, would kill people. Curiously although the right answer is clearly a sine curve (well, presumably) the model is a parabola, and oddly we're not told $H=2$ is the minimum height. The curve could conceivably go



though in fact the given answer is clear that this doesn't happen, and the model doesn't allow it either. Anyway:

a) $H(0) = 2 = a - b(-20)^2 = a - 400b$ —— ①

We don't know any other (t, H) points but we do know the maximum height is H.

So look for $\frac{dH}{dt} = 0$, and trust that 'since it's a parabola, theres only one such point:

$$\frac{dH}{dt} = -b(2t - 40) = 0$$

So $t = 20$.

And at this point $H(20) = a - b(0) = a$
$= 60$.

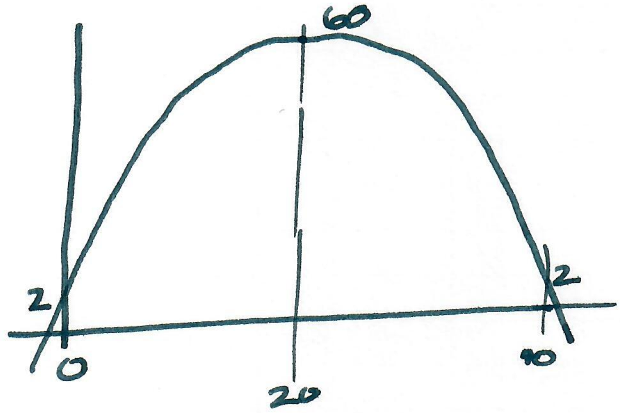
So $a = 60$ and from ①.

$$2 = 60 - 400b$$

$$b = \frac{58}{400} = \frac{14.5}{100} = 0.145.$$

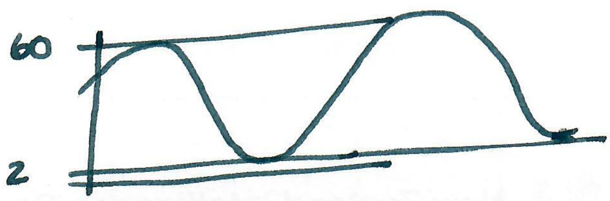
So the equation is $H = 60 - 0.145(t-20)^2$ ②

b) when $t = 40$, symmetry and ② tell us H is back to 2.
By now we can see what a stupid model this is:



(It might actually be a better model for an upward bungee stretch).

c) Now sanity comes in, and (lo!) we finally learn the initial point is (a) a minimum and (b) a turning point - though we still have to work out how to make the equation fit (b).



Given the curve is $H = 29\cos(9t + \alpha) + \beta$

we clearly need β to be $\frac{2+60}{2} = 31$,

and then we need α so that

$$H(0) = 2 = 29\cos(0+\alpha) + 31.$$

This means $\cos(0+\alpha) = -1$ or $\alpha = 180°$.

So $H = 29\cos(9t + 180)° + 31$.

d) well;-it wouldn't kill people.

- it follows the proper sine curve
- it has a minimum value of 2 for all t (and we know a journey is multiple circuits)
- it allows for multiple circuits (parabola doesn't)
- it starts with horizontal motion, consistent with most roller.coaster boarding platforms (though not, for instance, a funicular railway)

14) $(n+1)^3 - n^3 = \cancel{n^3} + 3n^2 + 3n + 1 - \cancel{n^3}$

$= 3n(n+1) + 1$

For any n, either n or $n+1$ is even – so the product $3n(n+1)$ must be even.

So $3n(n+1)+1$ must be odd

hence $(n+1)^3 - n^3$ is odd th ... as required.

15) $y = f(x) = \dfrac{7xe^x}{\sqrt{e^{3x}-2}}$

Using $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{vu' - uv'}{v^2}$; and $d(uv) = udv + vdu$:

$$f'(x) \quad \frac{dy}{dx} = 7\,\frac{\left(\sqrt{e^{3x}-2}\right)\left(xe^x + e^x\right) - xe^x \dfrac{1}{\sqrt{e^{3x}-2}}\left(3e^{3x}\right)\times\frac{1}{2}}{\left(e^{3x}-2\right)}$$

$$= 7\,\frac{\left(e^{3x}-2\right)\left(xe^x + e^x\right) - \frac{x}{2}e^x\,3e^{3x}}{\left(e^{3x}-2\right)^{\frac{3}{2}}}$$

$$= \frac{7e^x\left(\left(e^{3x}-2\right)(x+1) - \frac{3xe^{3x}}{2}\right)}{\left(e^{3x}-2\right)^{\frac{3}{2}}}$$

$$= \frac{7e^{x}\left(e^{3x}\left(x+1-\frac{3}{2}x\right) - 2(x+1)\right)}{\left(e^{3x}-2\right)^{3/2}}$$

$$= \frac{7}{2}e^{x}\ \frac{\left(e^{3x}(2-x) - 4x - 4\right)}{\left(e^{3x}-2\right)^{3/2}}$$

$$= \frac{7e^{x}\left(e^{3x}(2-x) - 4x - 4\right)}{2\left(e^{3x}-2\right)^{3/2}} \quad \text{——— } 1$$

as required, with $A = B = -4$.

b) with $x > \ln \sqrt[3]{2}$, $\quad e^{3x} > 2$

so $e^{3x} - 2$ is always > 0

So setting $f'(x) = 0$ and multiplying 1 by $2(e^{3x}-2)^{3/2}$:
(and also dividing by $7e^{x}$, also > 0:)

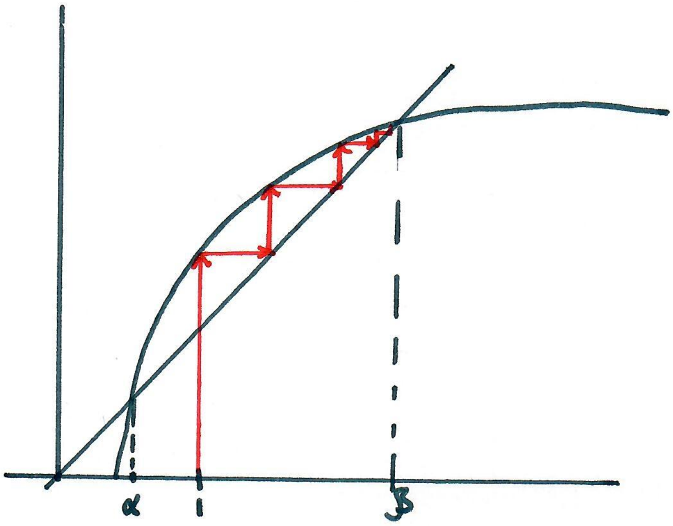
$$e^{3x}(2-x) - 4x - 4 = 0$$

$$2e^{3x} - x^{3x} - 4x - 4 = 0$$

$$-x\left(e^{3x}+4\right) = -2e^{3x} + 4$$

$$x = \frac{2e^{3x} - 4}{e^{3x}+4}$$

c)



$$x_{n+1} = \frac{2e^{3x_n} - 4}{e^{3x_n} + 4}$$

$$x_1 = 1$$

$$x_2 = \frac{2e^{3} - 4}{e^{3} + 4} = 1.502$$

$$x_3 = \frac{2e^{4.506} - 4}{e^{4.506} + 4} = 1.873$$

$$x_4 = \frac{2e^{5.619} - 4}{e^{5.619} + 4} = 1.957$$

$$x_5 = \frac{2e^{5.871} - 4}{e^{5.619} + 4} = 1.957$$

so this has converged sufficiently to say $\beta = 1.957$ to 3dp.

For α the staircase diagram doesn't appear to work (though it might if we tweaked it a bit):
instead, though, we're given $\alpha = 0.432$ to try:

$$x_1 = \frac{2e^{1.296} - 4}{e^{1.296} + 4} = 0.432 \text{ to } 3\text{dp}$$

This is converging slightly, but it's good to 3dp.

Short of this, I'm not sure what the question wants- though it seems to want something.