I really find statistics very difficult, and nasty.

1) a) AI used: 22 — 26% detected, 74% missed.

AI not used: 78 — 1% detected falsely, 99% correct as not AI.

~~So on 1~~

Assume on 100 pieces of work 22% use AI.

No. of pieces detected as AI $= 22 \times 0.26 = 5.72$
$+ 78 \times 0.01 = .78$ ①

$= 6.5$

So on 120 pieces of work this is

$6.5 \times \frac{120}{100} = 7.8$ or almost 8.

Likelihood of a false +ve = from ① $\frac{.78}{\cancel{5.72}\ 6.5}$

$= \cancel{.1333}$
0.12

This is approx 1 in 8, which is significant...

So there is a risk of unfair accusation.

b) 1% cheat

20% complete early

P (complete early) = 0.20

80% who cheat complete early.

P (complete early / cheat) = 0.80.

i) student cheats: p(student cheats) = 0.01

p(completes early) = 0.20

Are these indt? p(completes early | cheat) = 0.8

not equal, so they're not ind^t.

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or for independence,

Prob (cheat ∩ finish early) = 0.01×0.8 = 0.008

= Prob(cheat) × Prob(comp early)

0.01 × 0.2

= 0.002

not equal,

so not independent.

It seems if students are cheating they are four times as likely to complete early.

Of course this doesn't prove. What we want is pr(cheat | finish early) - which is Bayes...

ii) Prop. who cheat = 1% (prob = 0.01)

Prop who cheat and complete early = 80° (prob = 0.8)

So prop. of the whole = 0.01 × 0.8 = 0.008

or .8%.

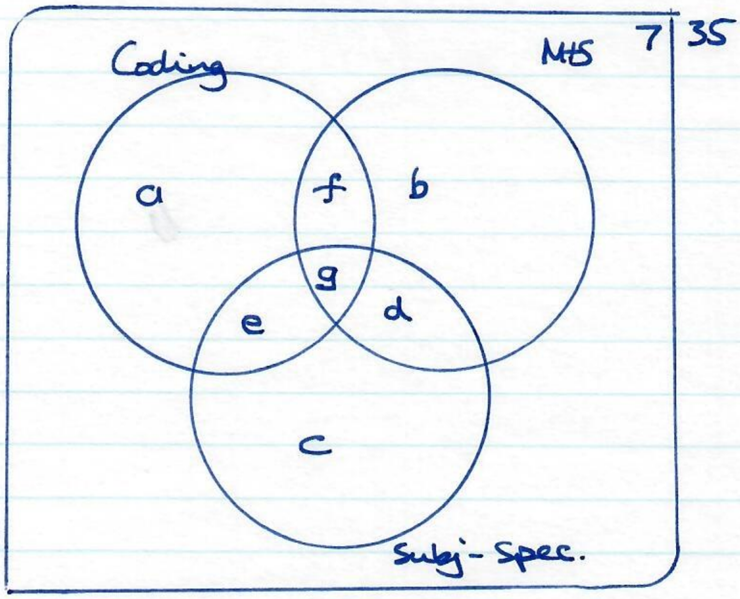
iii) Reasons for caution:

The data was collected from only one school

The data relates to only one standard.

(There may be other kinds of cheating.)

2)a) This looks challenging but it's basically a 3-set Venn diagram question where we have to work out the numbers. It would help if they'd written the question more carefully ... all the 'ands' get a bit confusing. And even then, a lot of it is 'shot in the dark' territory.



We're given:

$a+b+c+d+e+f+g = 35-7 = 28$ ①

$a+e+f+g = 21$ ②

$b+d+f+g = 16$ ③

$c+d+e+g = 6.$ ④

$d+e+f = 9$ ⑤

$f = 7$ ⑥

$c = 1$ ⑦

$\frac{3}{8}(a+b+c) = b$ ⑧

We also note: No variable can be <0, though 0 is possible. ⑨

All variables are integers ⑩

⑥ and ⑦ are surprisingly 'simple', but they're clearly meant to help.

use (2) + (3) + (4):

$$a+b+c+2(d+e+f)+3g = 21+16+6 = 43$$

use (5): $a+b+ \quad + 2 \times 9 + 3g = 43$

So $a+b \quad + 3g = 24$ —— (11)

Usefully this tells us $a+b$ is a multiple of 3.

Also from (8),

$$3a+3b+3 = 8b$$

$5b = 3(a+1)$ so $5b$ is a multiple of 3.

Also from (1), $a+b+1+9+g = 28$

$a+b+g = 18.$ —— (12)

So (11) − (12): $2g = 6$ $\underline{\underline{g=3.}}$

This means that in (11) $a+b = 15$
and (12) $5b = 3(a+1)$

Try values of b:

b	5b	3a+1	a
b = 0	0	0	×
1	5	5	×
2	10	10	×
3	15	15	4
4	20	20	×
5	25	25	×
6	30	30	9

So possibilities are $a=4$, $b=3$ —— (13)
or $a=9$, $b=6$. —— (14)

Check these in 2 and 3:

(4,3) $\quad 4 + e + 7 + 3 = 21$
$\quad\quad 3 + d + 7 + 3 = 16$ $\quad \Rightarrow \begin{cases} e = 7 \\ d = 3 \end{cases}$ $\quad$ 15

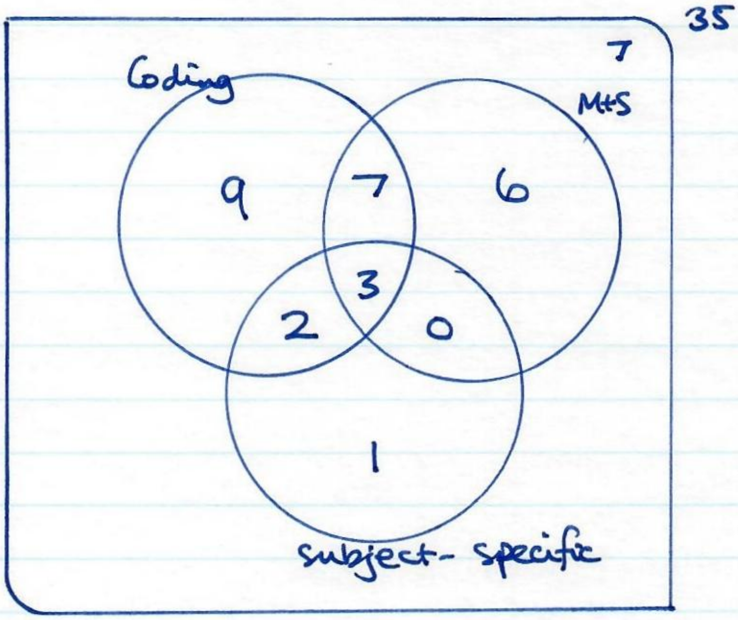
(9,6) $\quad 9 + e + 7 + 3 = 21$
$\quad\quad 6 + d + 7 + 3 = 16$ $\quad \Rightarrow \begin{cases} e = 2 \\ d = 0 \end{cases}$ $\quad$ 16

15 is contradicted by 5, but 16 is consistent with 5. So we conclude:

$a = 9$
$b = 6$
$c = 1$
$d = 0$
$e = 2$
$f = 7$
$g = 3.$

(Seems an incredibly long process for what is just the start of a question)

And the diagram is:



This checks with all the conditions specified.

i) People with all 3 skillsets = $g = 3$.

ii) People with only one of the skills = $\dfrac{a+b+c}{35}$

$$= \frac{16}{35} = \underline{\underline{0.457}}$$

iii) Among those who have M+S, those who also have Coding are 10 and those who also have SS are 3.

So the ratio of SS to Coding is $\frac{3}{10}$,

not $\frac{2}{1}$ which the claim would require.

You could also do this as:

$P(SS \mid M{+}S) = \frac{3}{16}$; $P(C \mid M{+}S) = \frac{10}{16}$

So the ratio of the two probabilities is $\frac{3}{16} \Big/ \frac{10}{16} = \underline{\underline{\frac{3}{10}}}$ etc.

b) Can't actually see the diagram, so I can't answer.

3) a) Game A: Probabilities are given by the usual values for dice

Sum	= 2 or 12	$\frac{1}{36}$
	= 3 or 11	$\frac{2}{36}$
	4 or 10	$\frac{3}{36}$
	5 or 9	$\frac{4}{36}$
	6 or 8	$\frac{5}{36}$
	7	$\frac{6}{36}$

So the probability of 5–10 incl. is

$$= \frac{1}{36}(4 + 5 + 6 + 5 + 4 + 3)$$

$$= \frac{27}{36} = \underline{\underline{\frac{3}{4}}}$$ Prob. of winning in one game.

Prob. of losing $= \frac{1}{4}$, so prob of winning at least once in 3 games $= 1 - \left(\frac{1}{4}\right)^3 = \underline{\underline{0.984375}}$

b) This is a probability tree, though it looks like a killer to draw. See next sheet.

	Prob (/36)		2nd roll.
2	1	lose	
3	2	lose	
4	3		win $\frac{3}{36}$ lose $\frac{6}{36}$ stay $\frac{27}{36}$
5	4		win $\frac{4}{36}$ lose $\frac{6}{36}$ stay $\frac{26}{36}$
6	5		" $\frac{5}{36}$ — $\frac{6}{36}$ — $\frac{25}{36}$
7	6	win	
8	5		$\frac{5}{36}$ $\frac{6}{36}$ $\frac{25}{36}$
9	4		$\frac{4}{36}$ $\frac{6}{36}$ $\frac{25}{36}$
10	3		$\frac{3}{36}$ $\frac{6}{36}$ $\frac{27}{36}$
11	2	lose	
12	1	lose	

$$Pr(\text{win}) = \frac{1}{6}$$

$$Pr(\text{lose}) = \frac{1}{6}$$

$$Pr(\text{still in}) = \frac{4}{6}.$$

~~It's not clear whether the point stays the same in successive rolls or changes with each roll. though actually it doesn't matter since we only really do two rolls.~~

Adding up the 'loses' in 2nd roll, these are always $\frac{6}{36}$.

So the prob. of losing before a 3rd roll

$= Pr(\text{lose at roll 1})$ $\frac{1}{6}$

$+ Pr(\text{stay at roll 1 but lose at roll 2}) = \frac{4}{6} \times \frac{6}{36} = \frac{1}{9}$

$= \frac{3}{18} + \frac{2}{18} = \frac{5}{18} = \underline{\underline{0.2778}}$