Edexcel Pure Maths Paper 1: A Level 2022

1 $P(-2,-5)$ lies on $y=f(x)$. Transformed to:

a) under $y=f(x)+2$ $(-2,-3)$

b) under $y=|f(x)|$ $(-2,5)$

c) under $y=3f(x-2)+2$ $(0,13)$

(a) is simple

(b) involves reflection in x-axis

(c) to find this point, take $x=0$

then y follows.

2 $f(x) = (x-4)(x^2-3x+k)-42$

$x+2$ is a factor, so $f(-2)=0$

substituting:

$$f(-2) = -6\left((-2)^2-3.-2+k\right)-42$$

$$= -6(4+6+k)-42$$

So
$$42 = -6(10+k)$$
$$7 = -10-k$$
$$k = -10-7$$
$$= -17.$$

3. Circle $x^2 + y^2 - 10x + 16y = 80$

Standard equations for the centre of a circle $x^2 + y^2 - 2px - 2pq = r$ are (p, q): so the centre is $(5, -8)$

Similarly the radius is $\sqrt{p^2+q^2+r}$

$$= \sqrt{25+64+80}$$

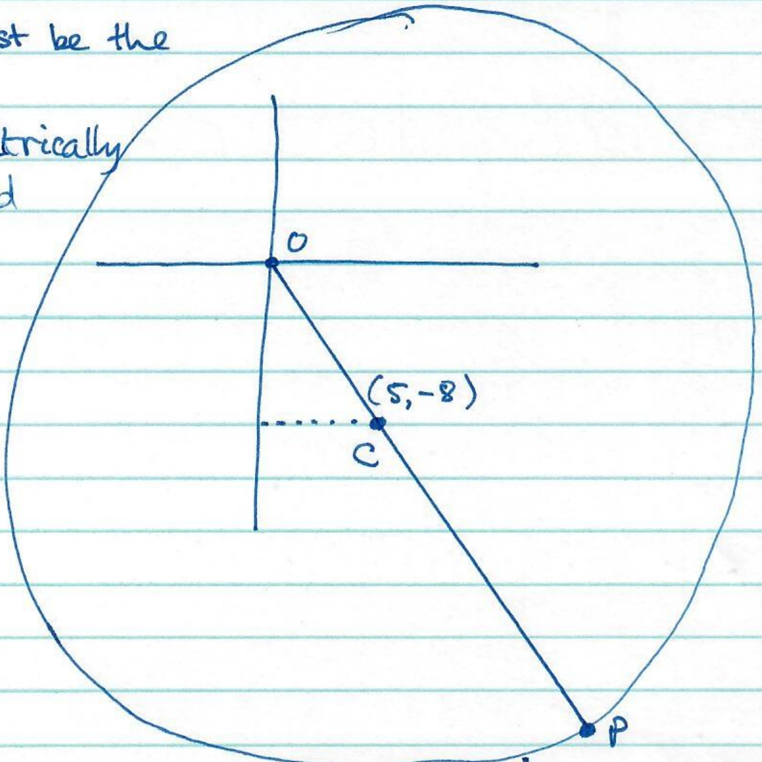
$$= \sqrt{169} = 13$$

(or express it as a sum of squares:

$$(x-5)^2 - 25 + (y+8)^2 - 64 = 80$$

$$(x-5)^2 + (y+8)^2 = 169 \quad \text{etc.})$$

b) P must be the point diametrically opposed to O:



$CP = 13$

By Pythagoras –

$$OC = \sqrt{5^2 + 8^2}$$

$$= \sqrt{25+64}$$

$$= \sqrt{89}.$$

So $OP = 13 + \sqrt{89}$

4 a) The sum is exactly the definition of the integral function $\int_{2 \cdot 1}^{6.3} \frac{2}{x}\,dx$

Evaluating the integral,

$$\int_{-2.1}^{6'3} \frac{2}{x}\,dx = 2\int_{2 \cdot 1}^{6.3} \frac{1}{x}\,dx$$

$$= 2\ln x \Big|_{2.1}^{6.3}$$

$$= 2\left(\ln 6.3 - \ln 2.1\right)$$

$$= 2\ln\frac{6.3}{2.1}$$

$$= 2\ln 3.$$

$$= \ln 3^2$$

$$= \ln 9$$

So $k = 9$ as required.

5a $h^2 = at + b \qquad 0 \le t < 25$

$h(2) = 2.60\text{m}$
$h(10) = 5.10\text{m}$

Substituting in:

$2a + b = 2.6^2 = 6.76 \qquad (1)$

$10a + b = 5.1^2 = 26.01 \qquad (2)$

(2) − (1): $8a = 26.01 - 6.76 = 19.25$

$a = 2.406 \quad (2.41 \text{ to } 3\text{sf})$

in (1): $b = 6.76 - 2 \times 2.406$
$= 1.9475 \quad (1.95 \text{ to } 3\text{sf})$

5b The model suggests that

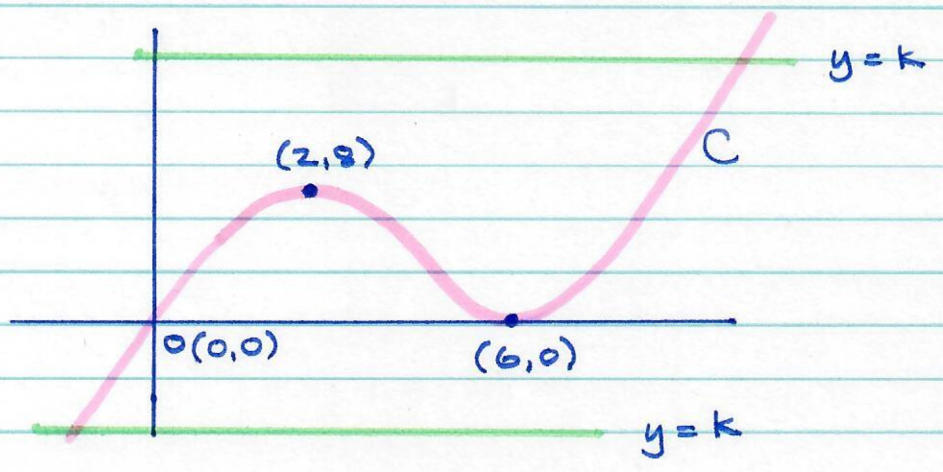
$h(20) = \sqrt{2.41 \times 20 + 1.95}$

$= \sqrt{50.15}$

$= 7.08 \quad (3\text{ sf})$

which indicates very close agreement with the actual height of 7m.

6



a) $f'(x) < 0$ when the curve is decreasing,

i.e. when $2 < x < 6$

b) If $y = k$ intersects C at only one point

then $k > 8$ or $k < 0$ (note not equalities)

i.e. k is defined in set notation

by $\{k : 8 < k, \; k < 0\}$

c) Since $f(x)$ has a single root at $x = 0$ and a double root at $x = 6$,

it must have factors $(x-6)^2$ and x.

So let $f(x) = a x (x-6)^2$ where a is to be found.

Substituting in for the point $(2, 8)$:

$$8 = a \cdot 2 \cdot (2-6)^2 = a \cdot 2 \cdot 16$$
$$= 32a$$

So $a = \frac{1}{4}$ So $f(x) = \frac{x}{4}(x-6)^2$

7. i Assume neither of p and q is even:

so $p = 2x + 1$
$q = 2y + 1$

Then $pq = (2x+1)(2y+1)$

$= 4xy + 2x + 2y + 1$

$=$ [even number] $+1$

$=$ [odd number]

This contradicts the condition that pq is even, so we have proved by contradiction that at least one of p and q must be even

(as required)

7 ii $x < 0$ ①

$(x+y)^2 < 9x^2 + y^2$ ②

Expand ②:

$$x^2 + 2xy + y^2 < 9x^2 + y^2$$

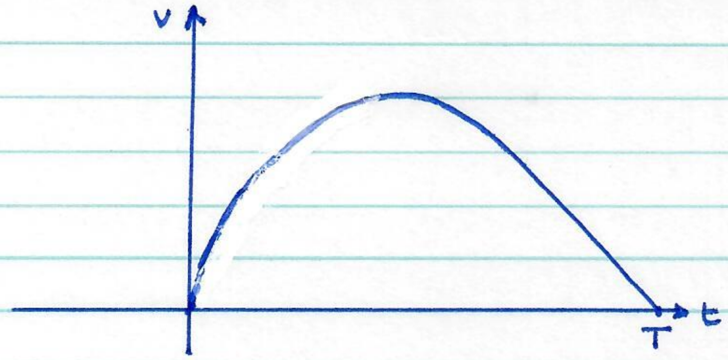
$$2xy < 8x^2$$

$$xy < 4x^2$$

Divide through by x, but noting $x < 0$ so the inequality is reversed: $y > 4x$

as required.

8



$$v = (10 - 0.4t)\ln(t+1)$$

8a At the point $(T, 0)$:

$$0 = (10 - 0.4T)\ln(T+1)$$

$\ln(T+1)$ is not 0, so

$$10 - 0.4T = 0$$

$$0.4T = 10$$

$$T = \frac{10}{0.4} = 25 \text{ sec}$$

8b Maximum speed is when $\frac{dv}{dt} = 0$, so:

$$\frac{dv}{dt} = \frac{d}{dt}\left[(10-0.4t)\ln(t+1)\right]$$

= (differentiating the product)

$$(10-0.4t)\frac{1}{t+1} - 0.4\ln(t+1)$$

So $$0 = \frac{10-0.4t}{t+1} - 0.4\ln(t+1)$$

$$0 = 10 - 0.4t - 0.4(t+1)\ln(t+1)$$

$$0.4t + 0.4t\ln(t+1) = 10 - 0.4\ln(t+1)$$

$$t(0.4 + 0.4\ln(t+1)) = 10 - 0.4\ln(t+1)$$

$$t = \frac{10 - 0.4\ln(t+1)}{0.4(1+\ln(t+1))}$$

$$= \frac{25 - \ln(t+1)}{1+\ln(t+1)}$$

$$= \frac{26}{1+\ln(t+1)} - (1+\ln(t+1))$$

$$= \frac{26}{1+\ln(t+1)} - 1 \quad \text{as required}$$

(That was a hard slog)

8c

$t_1 = 7$ (And so was this)

$$t_2 = \frac{26}{1+\ln(7+1)} - 1 = 7.443$$

$t_3 = 7.297$

$t_4 = 7.344$

$t_5 = 7.329$

$t_6 = 7.333$

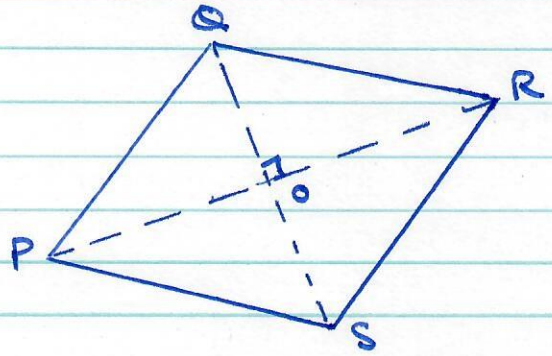
$t_7 = 7.332$

$t_8 = 7.333$

$t_9 = 7.333$

It seems fair to say the time is 7.333 sec (3dp)

9.



$$\overrightarrow{PQ} = 2\underline{i} + 3\underline{j} - 4\underline{k}$$

$$\overrightarrow{QR} = 5\underline{i} - 2\underline{k}$$

a We know PQRS is a parallelogram, so to show it is a rhombus we need only show that $|\overrightarrow{PQ}| = |\overrightarrow{QR}|$:

$$|\overrightarrow{PQ}| = \sqrt{2^2+3^2+4^2}$$
$$= \sqrt{4+9+16}$$
$$= \sqrt{29}$$

$$|\overrightarrow{QR}| = \sqrt{5^2+0^2+2^2}$$
$$= \sqrt{25+4}$$
$$= \sqrt{29}$$

So PQRS is a rhombus.

b Since PQRS is a rhombus its diagonals meet at right angles and at their midpoints: so the area =

$$4 \times \frac{|\overrightarrow{PR}|}{2}\frac{|\overrightarrow{QS}|}{2} \times \frac{1}{2}$$ (4 x area of the small triangles)

$$= \frac{1}{2}|\overrightarrow{PR}||\overrightarrow{QS}|$$

$$\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + \begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 7 \\ 3 \\ -6 \end{pmatrix}$$ (using ( ) notation)

$$\text{So } |\overrightarrow{PR}| = \sqrt{7^2+3^2+6^2} = \sqrt{49+9+36}$$

$$= \sqrt{94}$$

$$\overrightarrow{QS} = \overrightarrow{QP} + \overrightarrow{PS}$$

$$= -\overrightarrow{PQ} + \overrightarrow{QR}$$

$$= \begin{pmatrix} -2 \\ -3 \\ 4 \end{pmatrix} + \begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ 2 \end{pmatrix}$$

$$\text{So } |\overrightarrow{QS}| = \sqrt{3^2+3^2+2^2} = \sqrt{9+9+4}$$

$$= \sqrt{22}$$

So the exact area of PQRS

$$= \frac{1}{2}.\sqrt{94}.\sqrt{22} = \frac{2}{2}\sqrt{47}.\sqrt{11}$$

$$= \underline{\underline{\sqrt{517}}}$$

10. $N_b = 45 + 220e^{0.05t}$ (N is <u>thousands</u>)

a) At the start of the study $t=0$

so $N_b(0) = 45 + 220e^{0.05 \cdot 0}$

$= 45 + 220e^{0}$

$= 45 + 220.1$

$= \underline{\underline{265}}$ thousands of bees.

b) The rate of increase is the gradient,

ie derivative $\frac{dN_b}{dt}$

This equals $220 \times 0.05\, e^{0.05t}$

$= 11e^{0.05t}$

So the rate when $t=10$ is $11e^{0.05 \times 10}$

$= 18.13$ or approximately 18 thousand per year.

c) $N_w = 10 + 800e^{-0.05t}$

We require $N_b = 45 + 220e^{0.05T} = 10 + 800e^{-0.05T} = N_w$

So $35 = 800e^{-0.05T} - 220e^{0.05T}$

Let $y = e^{0.05T}$

then $35 = \frac{800}{y} - 220y$

So $\quad 35y = 800 - 220y^2$

$$220y^2 + 35y - 800 = 0$$

$$44y^2 + 7y - 160 = 0$$

$$y = \frac{-7 \pm \sqrt{49 + 4.44.160}}{2.44}$$

$$= \frac{-7 \pm 167.96}{88}$$

$$= -1.988 \quad \text{(discard)}$$

or $\quad 1.829$

(Discard the negative solution since this cannot be an $e^{0.05T}$ term)

$$y = 1.829 = e^{0.05T}$$

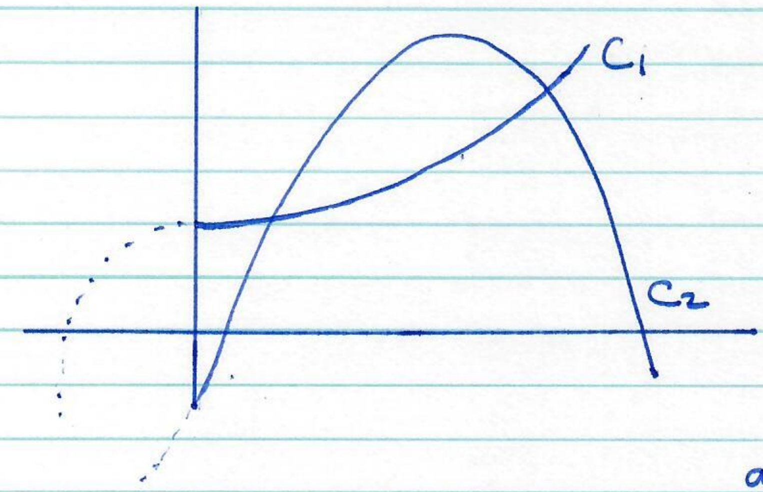
So $\quad 0.05T = \ln 1.829$

$$T = \frac{\ln 1.829}{0.05}$$

$$= 12.08 \text{ years}$$

So the wasp and bee populations will be equal at 12.08 years.

11.



a) To verify the curves intersect at $x = \frac{1}{2}$ we need only check the y values are the same

$$C_1: \quad y = 2x^3 + 10 = 2 \cdot \left(\tfrac{1}{2}\right)^3 + 10$$

$$= \frac{2}{8} + 10 = \frac{41}{4}$$

$$C_2: \quad y = 42x - 15x^2 - 7$$

$$= \frac{42}{2} - 15\left(\tfrac{1}{2}\right)^2 - 7$$

$$= 21 - \frac{15}{4} - 7$$

$$= 14 - \frac{15}{4}$$

$$= \frac{56 - 15}{4} = \frac{41}{4}$$

So the curves do intersect when $x = \frac{1}{2}$, at the point $\left(\frac{1}{2}, \frac{41}{4}\right)$.

11b To find P, again set the y values equal:

$$2x^3+10 = 42x-15x^2-7$$

so $$2x^3+15x^2-42x+17=0$$

we know $x=\frac{1}{2}$ is a solution, so this has a factor $(2x-1)$. Factorising gives

$$(2x-1)(x^2+8x-17)=0$$

So P is one of the roots of the quadratic term.

These are $$x = \frac{-8 \pm \sqrt{64+4.17}}{2}$$

One of these is negative so we reject it since the question is set for $x>0$:

the remaining root is $$x = \frac{-8+\sqrt{64+68}}{2}$$

$$= -4+\sqrt{16+17}$$

$$= \underline{\underline{\sqrt{33}-4}}$$

(The other root does work of course: but the curves are rather hard to plot. They meet at $(-9.7, -1840)$)

12
$$\int_1^{e^2} x^3 \ln x \, dx$$

use differentiation of a product:

$$\frac{d}{dx} uv = uv' + u'v$$

So
$$uv = \int uv' + \int u'v$$

i.e
$$\int uv' = uv - \int u'v$$

Let $u = \ln x$ and $v = \frac{x^4}{4}$

Then (noting limits)
$$\int x^3 \ln x \, dx = \frac{x^4}{4} \ln x - \int \frac{1}{x} \cdot \frac{x^4}{4} dx$$

$$= \frac{x^4}{4} \ln x - \int \frac{x^3}{4} dx$$

$$= \left[\frac{x^4}{4} \ln x - \frac{x^4}{16} \right]_1^{e^2}$$

$$= \left[\frac{x^4}{4} \left(\ln x - \frac{1}{4} \right) \right]_1^{e^2}$$

$$= \frac{e^8}{4} \left(\ln e^2 - \frac{1}{4} \right) - \frac{1}{4} \left(\ln 1 - \frac{1}{4} \right)$$

$$= \frac{e^8}{4} \left(2 - \frac{1}{4} \right) - \frac{1}{4} \left(0 - \frac{1}{4} \right)$$

$$= \frac{7}{16} e^8 + \frac{1}{16}$$

13i Assume (since it's not spelled out):

$$a_1 = a$$
$$a_2 = a + d$$
$$a_n = a + (n-1)d$$

$$S'_1 = a_1$$
$$S'_2 = a_1 + a_2$$
$$S'_n = \sum_1^n a_i$$

Then $S'_n = S'_{n-1} + a_n$

$= S'_{n-1} + a + (n-1)d.$

If this agrees with the formula given, then we can declare the formula is proved by induction.

Substituting:

$$LHS = S_n = \frac{n}{2}[2a + (n-1)d]$$

$$RHS = \cancel{S}\ S_{n-1} + a + (n-1)d$$

$$= \frac{n-1}{2}[2a + (n-2)d] + a + (n-1)d$$

$$= \frac{n-1}{2} \cdot 2a + \frac{2a}{2} + \frac{(n-1)(n-2)d + 2(n-1)d}{2}$$

$$= \frac{2na}{2} + \frac{n(n-1)d}{2}$$

$$= \frac{n}{2}\left[2a + \frac{(n-1)d}{2}\right] = LHS$$

So the induction is proved, and the formula is obviously correct for S_1.

Thus $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ as required.

ii) In n weeks James will have saved S_n pounds where $a = 10$ and $d = -0.80$

So $S_n = 64$

$= \frac{n}{2}\left(20 + (n-1)(-0.8)\right)$ note the – sign

$128 = 20n - n(n-1)0.8$

~~$0.8n^2 + 20n - 0.8n + 128 = 0$~~

$160 = 25n - n(n-1) = 26n - n^2$

a) $n^2 - 26n + 160 = 0$

b) $(n-10)(n-16) = 0$

so $n = 10$ or 16 (weeks)

c) James will have saved enough money in 10 weeks; if he continues saving his balance will rise higher – but only just. Since he is saving less and less each week the amount he 'saves' will eventually fall below £0, and he will start eating into his savings – bringing them back down to £64

14a) Given $2\sin(x-60°) = \cos(x-30°)$

It would be nice to do this simply by shifting things a bit by, say 30° here and there - but no, we'll have to do expansions:

$$2(\sin x\cos 60° - \sin 60°\cos x)$$

$$= \cos x\cos 30 + \sin x\sin 30°$$

$$\therefore \quad 2\sin x\cos 60° - 2\sin 60°\cos x$$

$$= \cos x\cos 30° + \sin x\sin 30°$$

But $\cos 30° = \sin 60°$ and $\cos 60° = \sin 30°$

So $\quad 2\sin x\cos 60° - 2\sin 60°\cos x$

$$= \cos x\sin 60° + \sin x\cos 60°$$

$$\therefore \quad \sin x\cos 60° - 3\sin 60°\cos x = 0$$

$$\sin x\cos 60° = 3\cos x\sin 60°$$

$$\frac{\sin x}{\cos x} = \frac{3\sin 60°}{\cos 60°}$$

$$= \frac{3\frac{\sqrt{3}}{2}}{\frac{1}{2}} = 3\sqrt{3}$$

as required.

14b If $2\theta = x - 60°$, i.e $x = 2\theta + 60°$

then $2\theta + 30° = x - 30°$

So from part (a) we can say

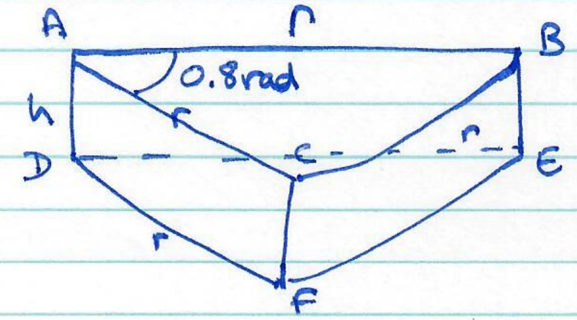
$$\tan(2\theta + 60°) = 3\sqrt{3}$$

so $2\theta + 60° = 79.1°$ is the principal solution.

$$2\theta = 19.1°$$
$$\theta = 9.6°$$

in the range $0 \leq \theta \leq 180°$ $\theta = 9.6° + 90° = 99.6°$ is also a solution

15



a) Volume of the toy $= h \times$ Area of top

$$= h \times \pi r^2 \times \frac{0.8}{2\pi}$$

$$= \frac{4}{10} h r^2$$

We know $\frac{4}{10} h r^2 = 240 \text{ cm}^2$

$$h r^2 = 600 \text{ cm}^2$$

a) S Surface area = 2 × top surface
+ 2 × radial edges
+ 1 × circumferential edge

$$= 2\pi r^2 \times \frac{0.8}{2\pi} + 2rh + 2\pi r \frac{0.8}{2\pi} h$$

$$= 0.8r^2 + 2rh + 0.8rh$$

$$= 0.8r^2 + 2.8rh$$

but $h = \frac{600}{r^2}$ from ①

So the surface area $S = 0.8r^2 + 2.8r.\frac{600}{r^2}$

$= 0.8r^2 + \frac{1680}{r}$ as required.

b) $$S = 0.8r^2 + \frac{1680}{r}$$

$$\frac{dS}{dr} = 2 \times 0.8r - \frac{1680}{r^2}$$ = 0 at a stationary point.

At such a point $1.6r = \frac{1680}{r^2}$

$$r^3 = \frac{1680}{1.6} = 1050$$

$$\underline{\underline{r = 10.16}}$$

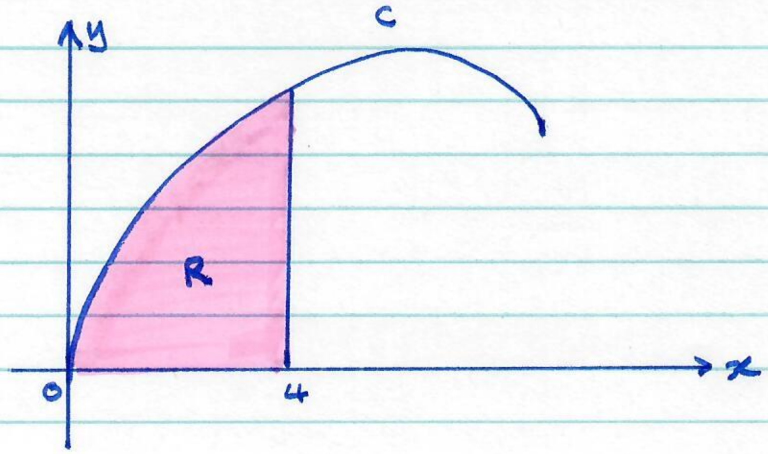
c) To show this is a minimum we can show $\frac{d^2S}{dr^2}$ is positive:

$$\frac{d^2S}{dr^2} = 1.6 + \frac{2 \cdot 1680}{r^3}$$

We don't need to calculate this to see that it's positive - it always is.

So this value of r gives the minimum surface area.

16



$x = 8\sin^2 t \qquad 0 \leq t \leq \frac{\pi}{2}$
$y = 2\sin 2t + 3\sin t$

Area $R = \int y\,dx$

$=$ (for parametric equations)

$\int y \frac{dx}{dt}\,dt$ between suitable bounds.

The point O is given by $t = 0$

When $x = 4$, $\quad 8\sin^2 t = 4 \quad$ so $\sin^2 t = \frac{1}{2}$

$$\sin t = \frac{1}{\sqrt{2}}$$

$$\text{so} \quad t = \frac{\pi}{4} \quad (45°)$$

So the bounds are 0 and $\frac{\pi}{4}$.

$$\frac{dx}{dt} = 2.8 \sin t \cos t = 16 \sin t \cos t$$

$$\text{So area } R = \int_0^{\frac{\pi}{4}} 16 \sin t \cos t \,(2 \sin 2t + 3 \sin t)\, dt$$

(where $a = \frac{\pi}{4}$ as required)

$$= \int_0^{\frac{\pi}{4}} 16 \sin t \cos t \,(4 \sin t \cos t + 3 \sin t)\, dt$$

(replacing the $\sin 2t$ term)

$$= \int_0^{\frac{\pi}{4}} 64 \sin^2 t \cos^2 t + 48 \sin^2 t \cos t \, dt$$

Now use the identities $\cos 2x = \cos^2 x - \sin^2 x$, $1 = \cos^2 x + \sin^2 x$

to give:

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

Substituting into the integral.

$$\int_0^{\frac{\pi}{4}} 64\underbrace{\left(\frac{1-\cos 2t}{2}\right)\left(\frac{1+\cos 2t}{2}\right)}_{\text{some terms drop out}} + 48\sin^2 t\cos t\,dt$$

$$= \int_0^{\pi/4} 64\left(\frac{1}{4} - \frac{\cos^2 2t}{4}\right) + 48\sin^2 t\cos t\,dt$$

$$= \int_0^{\pi/4} 16(1-\cos^2 2t) + 48\sin^2 t\cos t\,dt$$

Again use the earlier identity:

$$\cos^2 2t = \frac{1+\cos 4t}{2}$$

$$\int_0^{\pi/4} 16\left(1-\frac{1}{2}-\frac{\cos 4t}{2}\right) + 48\sin^2 t\cos t\quad dt$$

$$= \int_0^{\pi/4} 8 - 8\cos 4t + 48\sin^2 t\cos t\quad dt$$

$$= \left[8t - \frac{8\sin 4t}{4} + 48\frac{\sin^3 t}{3}\right]_0^{\pi/4}$$

$$= 8\frac{\pi}{4} - \cancel{\frac{8\sin\pi}{4}} + 16\sin^3\left(\frac{\pi}{4}\right)$$

$$\cancel{-8\cdot 0} - \cancel{8\frac{\sin 0}{4}} + \cancel{16\sin^3(0)}$$

$$= 2\pi + 16\left(\frac{1}{\sqrt{2}}\right)^3 = \underline{2\pi + 4\sqrt{2}}$$